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The Values of Information in Some Nonzero Sum Games

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Abstract: The paper considers nonzero-sum games in which the players' utility functions are not known with certainty. It tries to clarify the various definitions of information value in such games, provides the reader with a class of games with simple solutions, and presents experimental results corroborating the theoretical analysis.

1. Introduction

Consider nonzero sum games in which the players' utility functions are not known with certainty. Such games are usually modelled as games of incomplete information [*Harsanyi*]. In these games the utility functions are determined by a chance move, about which the players are partially informed. In this context, the present paper investigates the consequences on the players' payoffs, of changing their state of information on the outcome of the chance move. The following questions will be debated:

1. Is information always valuable?
2. Is it better to acquire information secretly or in front of others?
3. Is private information better than public information?

Information value theory is a rather well known subject in classical decision theory [*Raiffa*]. There, information is always valuable. More recently there has been much research on zero sum games of incomplete information and the basic result for these games shows that information is also always valuable [*Ponssard*]. Now for non zero sum games, the situation seems much more complicated, some studies indicate that information may become detrimental [*Basar/Ho*][3]). It may also be interesting to note that much earlier *Schelling* [1960, 146–150] arrived at the same conclusion but in a somewhat different context. He investigated the problem of communication between the players, whereas the subject considered here is a problem of information about some incertain event depending on nature.

The contribution of this paper may be seen as follows:

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[2]) Prof. *J.P. Ponssard*, Centre de Recherche en Gestion. École Polytechnique, 17, Rue, Descartes, F-75005 Paris.

[3]) In a private correspondance Shapley pointed out the fact that in a bargaining situation the Nash solution may be more favorable to the players in the presence of an incertain event than if they become aware of the outcome of this event.

(i) It tries to clarify the various definitions of information value for decision problems with multiple decision makers.
(ii) It provides the reader with a class of games with simple solutions[4]) in which all possibilities about the value of information may occur.
(iii) It presents experimental results which corroborate the theoretical analysis.

2. Preliminary Considerations on the Value of Information

To study information problems in competitive situations, we shall distinguish different types of information that one player may receive in a game.

Case 1: "secret" information
In this case, one player acquires the information, but the other players are ignorant of this fact. Note that the resulting situation cannot be analyzed as a game since the rules are not known to all players. Nevertheless one may assume that the uninformed players will not modify their behaviour. This assumption could be particularly unreasonable if the game was dynamic (in extensive form), since then the uninformed players would soon realize that they are playing a different game.

Case 2: "private" information
In this case, one player acquires the information and, though he is the only one informed, this fact is known to all players. Then, information may have several effects which cannot take place with secret information. First, the acquisition of information may give the opportunity to the informed player to use threats against the uninformed players without even using his information as such. Second, the uninformed players may modify their own behaviour and it is not clear whether this would benefit the informed player.

Case 3: "public" information
In this case, all players acquire the information and this is known to everybody.

At first sight, one could expect that secret information would be more valuable than private information, which in turn, would be more valuable than public information[5]). But this is not true. The following class of games shows that one may have all possible configurations.

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[4]) That is using only dominance considerations. Thus our conclusions cannot be attributed to the weakness of the solution concept.

[5]) This is of course the case in zero sum games as a direct consequence of the monotonicity of the value with respect to the strategy space.

3. The Values of Information in a Simple Class of Games

3.1 One Shot Games

Consider the following class of two-person games with incomplete information:

chance move

prob = 1/2 prob = 1/2

$$A_1 = \begin{array}{c} \\ \alpha \\ \beta \end{array}\begin{array}{cc} a & b \\ \left[\begin{array}{c}(a_{11}, a_{11}) \\ (a_{12}, a_{21})\end{array}\right. & \left.\begin{array}{c}(a_{21}, a_{12}) \\ (a_{22}, a_{22})\end{array}\right] \end{array} \qquad A_2 = \begin{array}{c} \\ \alpha \\ \beta \end{array}\begin{array}{cc} a & b \\ \left[\begin{array}{c}(a_{12}, a_{12}) \\ (a_{11}, a_{22})\end{array}\right. & \left.\begin{array}{c}(a_{22}, a_{11}) \\ (a_{21}, a_{22})\end{array}\right] \end{array}$$

Chance selects one of the two bimatrices A_1, A_2, each one with probability 1/2. The players' states of information about the outcome of this chance move will be varied. Then, player 1 and player 2 have the choice between two actions (α, β) and (a, b) respectively.

Note that each bimatrix is symmetric (the players are interchangeable) and that one goes from one bimatrix to the other by permuting the two columns and the two rows.

Furthermore, the following restrictions on the parameters are made:

(i) $a_{11} > a_{12}$ and $a_{21} > a_{22}$,
(ii) $a_{11} + a_{12} > a_{22} + a_{21}$,
(iii) $a_{12} + a_{21} > a_{11} + a_{22}$.
Note that this implies $a_{11} > a_{12} > a_{22}$.

These assumptions greatly simplify the analysis of this class of games.

Consider the game in which no player is informed of the outcome of the chance move. Then, the obvious solution is (α, a) since the game is a pure coordination problem with (α, a) dominating any other outcome. Each player's expected payoff is $(a_{11} + a_{12})/2$.

Using (i), in the bimatrix A_1, α dominates β and a dominates b; whereas in A_2, β dominates α and b dominates a. This solves the game in *case of public information on the outcome of the chance move.* Each player's expected payoff is $(a_{11} + a_{21})/2$.

If player 2 is informed of the outcome of the chance move, then he should play the strategy a in A_1 and b in A_2 (by (i)). Now if this *information is acquired secretly*, player 1's choice remainds α and player 2's expected payoff is a_{11}. If it is *acquired privately*, player 1's best choice is β, (using (iii)), and player 2's expected payoff is a_{21}.

Then, the value of public information for player 2 is $(a_{21} - a_{12})/2$. His value of secret information is $(a_{11} - a_{12})/2$ and his value of private information is $a_{21} - (a_{11} + a_{12})/2$.

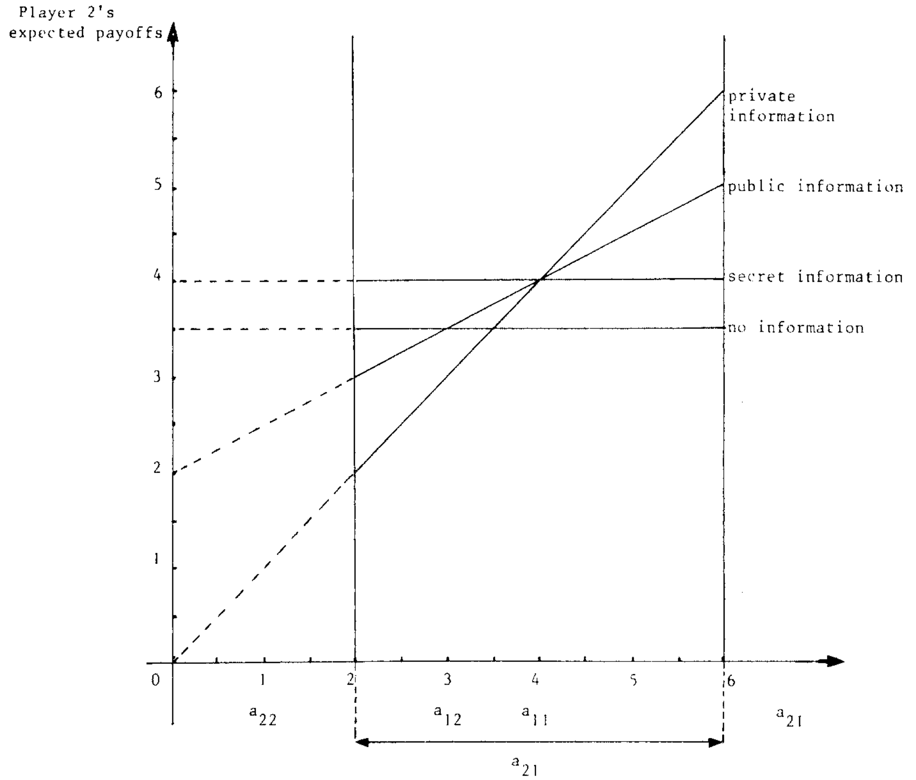

Fig.: Player 2's expected payoffs depending on the way he acquires information in the following game

1/2 1/2

$$\begin{bmatrix} 4,4 & a_{21},3 \\ 3,a_{21} & 1,1 \end{bmatrix} \quad \begin{bmatrix} 3,3 & 1,4 \\ 4,1 & a_{21},a_{21} \end{bmatrix}$$

Numerical values of parameters for this figure:

$a_{11} = 4; a_{12} = 3; a_{22} = 1; 2 = (a_{11} + a_{22}) - a_{12} \leqslant a_{21} \leqslant (a_{11} + a_{12}) - a_{22} = 6$

The graph above depicts the respective positions of these values when the parameter a_{21} is allowed to vary inside the constraints imposed by (i), (ii), and (iii).

Consequently, it is possible that the value of private information is negative, or smaller than the value of public information; it is also possible that secret information is less valuable than either private or public information.

Since the ranges of a_{21} for which these values are positive or negative do not seem to have intuitive interpretations there is little hope that a classification of games with respect to informational properties can be made.

The case in which the value of private information is negative deserves some more thought.

Player 2 should not have acquired this detrimental information. But if player 1 believes player 2 has this information, then the noncooperative assumption of the analysis leaves no way for him to demonstrate that he will not use it.

Remark it is even possible that player 1 may benefit from this situation, and so, has no incentive to play as if player 2 were not informed. Here is such an example:

chance move

prob = 1/2 prob = 1/2

$$A_1 = \begin{array}{c} \\ \alpha \\ \beta \end{array}\begin{array}{cc} a & b \\ \hline (2,2) & (2,2) \\ (3,1) & (0,0) \end{array} \qquad A_2 = \begin{array}{c} \\ \alpha \\ \beta \end{array}\begin{array}{cc} a & b \\ \hline (2,2) & (2,2) \\ (0,0) & (3,1) \end{array}$$

If nobody is informed the expected payoffs are (2,2) for players 1 and 2 respectively. Now if player 2 is privately informed, then, since a dominates b in A_1, and b dominates a in A_2, the payoff becomes (3,1).

It may be noted however that, for the class of symmetric games considered in this paper, if the value of private information is negative for the informed player, that is $a_{21} - (a_{11} + a_{12})/2 < 0$, the non-informed player cannot benefit from the situation:

Player 1's payoff in the game in which player 2 uses his private information, $(a_{12} + a_{21})/2$, is smaller than $(a_{11} + a_{12})/2$, his payoff in the game in which nobody is informed, since $a_{21} < a_{11}$.

This suggests that if such a game was iterated, then player 2 could play as if he was not informed and player 1 would not object. This is formally studied in the following section.

Remark 1. Our special class of games prevents us to observe more complicated features related to solution concepts.

Let us give an example:
The payoffs are given by:

chance move

prob = 1/2 prob = 1/2

$$A_1 = \begin{array}{c} \\ \alpha \\ \beta \end{array}\begin{array}{cc} a & b \\ \hline (0,9) & (-1,-4) \\ (-1,2) & (3,-3) \end{array} \qquad A_2 = \begin{array}{c} \\ \alpha \\ \beta \end{array}\begin{array}{cc} a & b \\ \hline (0,-3) & (-1,2) \\ (-1,-4) & (3,3) \end{array}$$

Thus, when nobody is informed, the expected payoffs are:

$$\begin{array}{c} \\ \alpha \\ \beta \end{array}\begin{array}{cc} a & b \\ \end{array}$$

$$\begin{matrix} & a & b \\ \alpha & (0,3) & (-1,-1) \\ \beta & (-1,-1) & (3,0) \end{matrix} .$$

If the second player is informed the strategy "a in the game A_1 and b in the game A_2" is a dominating strategy for him. Hence, player 1 has to play β, and the expected payoffs are (1, 5/2). But, in the game where nobody is informed, there are two Nash equilibrium points (α, a) and (β, b), which are not interchangeable.

If we assume that, it is (α, a), which will be actually played the value of private information is negative for player 2. But, if it is (β, b), this value is positive.

3.2 Iterated Games

We shall assume in this section that the games of the class defined in 3.1 are not played once, but are iterated an infinite number of times. Thus, we shall consider now the supergames [*Aumann; Luce/Raiffa*] associated with the games of 3.1. The payoffs in these supergames will be the average of the one shot payoffs.

We shall call *value of private information* for player 2 in the supergame, the difference between:

- his expected payoff in the supergame associated with the game in which he is privately informed and
- the expected payoff he obtains in the supergame associated with the game in which nobody is informed.

Let us recall that the set of payoffs of the supergame associated with a bimatrix game ($B = \| b_{ij} \|$, $C = \| c_{ij} \|$) is precisely the convex hull H of the vectors (b_{ij}, c_{ij}) of $\mathbf{R}^2$.

For an iterated bimatrix game (B, C) it is well known that an outcome belonging to the Pareto boundary of H can be achieved not only under cooperative behaviour but also under noncooperative behaviour using appropriate retaliatory strategies in case of deviation [*Aumann; Luce/Raiffa*]. Then, if one considers only such outcomes for the class of games defined in 3.1:

Theorem: The value of private information for player 2 in the supergame is positive.

Proof:

a) If in the one shot game, the information of player 2 is valuable for player 1 (i.e. $a_{11} < a_{21}$), one cannot find a pair of pure strategies of the one shot game in which player 2 is informed, such that the corresponding expected payoff for player 1 is greater than $(a_{12} + a_{21})/2$. Thus there is no Pareto point of the convex hull H of the expected payoffs of the one shot game which gives less than $(a_{11} + a_{12})/2$ to player 2, his payoff in the non-informed case.

b) If for the game played once, information is not valuable for player 1, (i.e. $a_{11} \geqslant a_{21}$), one cannot find a pair of pure strategies of the one shot game such that the corresponding payoff for player 1 is greater than $(a_{11} + a_{12})/2$. Thus, there is no Pareto point of H which gives less than $(a_{11} + a_{12})/2$ to player 2.

Remark 2. It is easy to check that, when information is valuable in the one shot game for player 2, the strategies: "play β at each stage", and "play a in A_1 and b in A_2 at each stage" are Nash equilibrium strategies of the supergame which are Pareto optimal. Moreover "play a in A_1 and b in A_2 at each stage" is also a MaxiMin strategy for player 2. Thus, it is likely that, when information if valuable for player 2, the players will play in the supergame as in the one shot game.

But if information is not valuable for player 2, it is not valuable for player 1. In this case it is likely that they will forget the information by playing: for player 1, "α as long as player 2 plays a, otherwise play β" and "a as long as player 1 plays α, otherwise play a in A_1, and b in A_2" for player 2. These strategies are in equilibrium and are Pareto optimal.

Remark 3. Theorem 1 relies mainly on the symmetry property of the class considered. If symmetry is not satisfied, as in the following example, the value of private information for player 2 in the supergame, may still be negative or null, whatever the Pareto point chosen as solution of the supergame.
The game is:

chance move

prob = 1/2   prob = 1/2

$$A_1 = \begin{array}{c} \\ \alpha \\ \beta \end{array}\begin{array}{cc} a & b \\ \left[\begin{array}{cc} (2,2) & (2,2) \\ (3,1) & (0,0) \end{array}\right] \end{array} \qquad A_2 = \begin{array}{c} \\ \alpha \\ \beta \end{array}\begin{array}{cc} a & b \\ \left[\begin{array}{cc} (2,2) & (2,2) \\ (0,0) & (3,1) \end{array}\right] \end{array}$$

Here, every point of the Pareto boundary of the convex hull of (0, 0), (2, 2), (3/2, 1/2) and (3,1), gives a payoff to player 2 less than or equal to 2, which is his payoff when he is not informed.

4. Some Experimental Results

A two-person game was experimented among a population of 25 students. Only two cases were studied: the case where no player is informed and the case where player 2 is privately informed.

Each student received a description of the rules of the game with the specification of the player he should act for. He had to play the experiment alone. His opponent was not physically determined. For each case, he was given the choice of selecting one pure strategy, or several ones among which he would be indifferent (no probabilities statements were asked for).

This experiment may be considered as a test for the solution concept used in this paper. Namely, player 2 has a different dominating action depending on the chance outcome. It should be noted that the value of private information for the game chosen is negative. We expected that the students would all play the theoretical solution.

The results of the experiment are presented as follows: the statistics of the selected pure strategies are gathered over all the students representing the same player. Then, the frequency of an outcome (i, j) of a game is computed by multiplying the observed frequencies of the pure strategies i and j. This is compared to the probability of the outcome (i, j) as it would result from the theoretical solution.
The game is:

chance

prob = 1/2 prob = 1/2

$$A_1 = \begin{array}{c} \\ \alpha \\ \alpha \end{array} \begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix} (0,4) & (4,0) \\ (1,1) & (1,1) \end{bmatrix} \end{array} \qquad A_2 = \begin{array}{c} \\ \beta \\ \beta \end{array} \begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix} (4,0) & (0,4) \\ (1,1) & (1,1) \end{bmatrix} \end{array}$$

theoretical solution — *experimental results*

a) no information:

$$\begin{array}{c} \\ \alpha \\ \beta \end{array} \begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix} 0,5 & 0,5 \\ 0 & 0 \end{bmatrix} \end{array} \qquad \begin{array}{c} \\ \alpha \\ \beta \end{array} \begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix} 0,48 & 0,48 \\ 0,02 & 0,02 \end{bmatrix} \end{array}$$

b) player 2 privately informed:

$$\begin{array}{c} \\ \alpha \\ \beta \end{array} \underset{A_1}{\begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \end{array}} \underset{A_2}{\begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \end{array}} \qquad \begin{array}{c} \\ \alpha \\ \beta \end{array} \underset{A_1}{\begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix} 0,23 & 0,01 \\ 0,73 & 0,03 \end{bmatrix} \end{array}} \underset{A_2}{\begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix} 0,01 & 0,23 \\ 0,03 & 0,73 \end{bmatrix} \end{array}}$$

The following observations may be made about the experimental results:
Almost all "player 2" students played their dominating strategy; however about 1/3 of "player 1" students did not change their behaviour in spite of the information revealed to player 2. They did not seem to understand that this new information was likely to modify player 2's strategy.

These partial results indicate that, even in this simple competitive situation, the students did not make a sharp distinction between secret and private information and were somewhat confused.

This remark may explain the answers to a questionnaire handed out to the students after the experiments. About 3/4 of them considered that the value of private information in competitive situations is always positive.

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