Selection in Dynamic Entry Games*

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This paper proposes a game theoretic framework to study dynamic competition with entry deterrence. Sufficient conditions are given such that the competition process results in the most efficient firm being eventually selected. We show this selection property for asymmetric firms to be the natural economic extension to the rent dissipation property obtained for symmetric firms. This framework is used to discuss previous models in which the selection property had not been studied (dynamic limit pricing and capacity renewal) or may fail to hold (quantity competition). *Journal of Economic Literature* Classification Numbers: C73, D43, L12. © 1997 Academic Press

1. INTRODUCTION

This paper is concerned with the efficiency properties of competition in a dynamic entry-deterrence framework. Consider a natural monopoly context, e.g. in which production involves large fixed costs. On the one hand, efficiency requires that a single firm operates. On the other hand, an unconstrained monopoly could adopt socially inefficient policies. Hence, an important issue is the extent to which the mere threat of entry by a competitor may ''discipline'' a monopoly and restore some efficiency without duplicating costs. Indeed, potential competition may force the incumbent to deviate from its optimal policy and instead adopt entry-preventing strategies (for surveys see Gilbert, 1989; Wilson, 1992). Such deviations reduce the incumbent's monopoly rent. In infinite horizon models of dynamic entry deterrence with symmetric firms, the no-entry condition takes the form of a simple recursive equation (Wilson, 1992): the incumbent's optimal entry-preventing policy (OEPP) is such that its rival's entry cost equals the total discounted rent that the latter would earn after entering; this rent is itself determined by the OEPP that the new incumbent would have to follow. A *rent dissipation property* obtains; the recursive equation implies that the incumbent's total discounted rent is bounded so that the flow profit has to go to zero as discounting decreases (Farrell, 1986). The implications for efficiency are ambiguous, although, and have triggered most past developments (Tirole, 1989).

Yet, an equally important issue is whether the competitive process selects the most efficient firm. To what extent can a historical incumbent deter entry by a more efficient competitor? Existing models cannot be used to tackle this question. They assume symmetric firms, which is key to formulating the rent dissipation argument. Despite their elegant simplicity, the direct technical extension of these approaches to asymmetric situations is not compelling. Eaton and Lipsey's (1980)'s argument relies on an intuitive, rational expectations reasoning which loses its bite with asymmetric firms. Maskin and Tirole's (1988)'s Markov equilibrium approach can lead to counterintuitive results; if the initial incumbent is the less efficient firm, it maintains indefinitely and earns the same rent as a more efficient firm would (Lahmandi-Ayed *et al.*, 1996). Finally, Ponssard's (1991)'s forward induction approach would require further refinements to handle the asymmetric case. To address this question, we thus resort to a finite horizon framework encompassing and extending the existing models. It builds on a finite series of Stackelberg stage games in which the leader's policy affects both its profit and its rival's entry cost. In dynamic settings, firms compete not only for the current demand but also for future incumbency and Stackelberg leadership. This is captured, under a reduced form, by endogenizing leadership; leadership in the next period's Stackel-

berg game rests with the current period's incumbent unless entry occurs, in which case it accrues to the successful entrant. Finally, the logic of entry-barriers is the same as for symmetric games; in each stage, the incumbent's OEPP sets the rival's entry cost at the level of the future rent its rival would earn if it entered, became the new incumbent, and had to follow its own OEPP.

The paper's central point is that the *selection property* of asymmetric games is the relevant economic extension to the *rent dissipation property* in symmetric ones, as well as its natural implication. The intuition is as follows. In the symmetric case, the incumbent's flow profit is driven down to zero by potential competition. In the asymmetric case, one firm has both a larger stage payoff for a given policy and a lower entry cost against a given policy. Under some sufficient assumptions on these functions, which we discuss, the less efficient firm's OEPP is more constrained than that of the more efficient one. Hence, its flow profit would have to be negative in the short-run so as to secure the positive incumbency rent earned in later stages. With a long horizon and low discounting, however, the losses accumulate to finally offset the future rent. Hence, the less efficient firm is better off exiting than preventing entry.

Although this analysis relies on a backward induction reasoning, it can be extended to some infinite horizon situations. As an illustration, a variant of the basic dynamic entry game is introduced in which firms are asymmetric during a finite phase, and then are symmetric forever. A similar selection property obtains. The analysis of this game is fruitful not only because it deals with a situation of economic relevance in which competitive (dis)advantages are only temporary. It also clarifies the role of our sufficient assumptions, providing an example in which the selection property holds under milder conditions.

Dynamic symmetric entry games play a central role in the discussion of contestability issues (Baumol *et al*., 1982; Fudenberg and Tirole, 1987). The introduction of asymmetry significantly contributes to this debate as it allows us to analyze the competition process not only as a disciplinary mechanism but as a selection device. The selection property requires more stringent conditions than the rent dissipation property. This is illustrated by means of three examples adapted from Ponssard (1991), Maskin and Tirole (1988), and Eaton and Lipsey (1980), in which entry is deterred through prices, capacities, and plant renewals. These examples also show that whether the selection property holds is bound to depend on the type of strategic variable considered.

The paper is organized as follows. The general framework is introduced in Section 2 and the main results are derived in Section 3. Section 4 applies this framework to the three examples. Section 5 concludes.

2. THE MODEL

We propose a model of dynamic competition between two firms in a natural monopoly context. In static Stackelberg entry games, the leader's policy tries to prevent entry by rivals in order to monopolize the demand. In dynamic entry games, the firms compete not just for the current demand but for future Stackelberg leadership as well, so as to monopolize future demand. Our model captures this double competition for demand and leadership in a reduced form. It is an extensive-form game of perfect information, consisting of a finite sequence of Stackelberg stage-games in which leadership is determined endogenously; leadership in the next period's Stackelberg game rests with the current period's incumbent unless entry occurs, in which case it accrues to the successful entrant. The game is diagrammed in Fig. 1.

There are two firms indexed by $i \in \{1, 2\}$. We consider games with a finite number of stages, N. For convenience, the stages are labelled backwards: N is played first, then stage $(N - 1)$ etc. In each stage, one firm is the leader. In the first stage, the leader is exogenous. In the other stages, it is determined endogenously (see below). We denote by $\Gamma_N(i)$ the N-stage game in which firm i is the initial leader.

Suppose that firm 1 is the leader in stage n; it can choose to play "*In*" or "*Out*." If firm 1 chooses "*In*," both firms play the following Stackelberg game, $G(1)$ (see below). If firm 1 chooses "*Out*," then firm 2 can choose between "*In*" and "*Out*." If it chooses "*In*" the firms play $G(2)$. If both firms choose "*Out*," they receive a payoff of zero in this stage and all remaining stages.

Game $G(i)$ is played as follows. First, firm i chooses a policy $x_i \in X$, where X is an interval in **R**. Second, firm j (with $j \neq i$) chooses a policy $x_j \in \{Out, In\}$. The payoffs v_i and v_j are as follows:

$$v_i(x_i, x_j) = \begin{cases} v_i(x_i) & \text{if } x_j = Out \\ d_i(x_i) < 0 & \text{if } x_j = In; \end{cases}$$

$$v_j(x_i, x_j) = \begin{cases} 0 & \text{if } x_j = Out \\ -C_j(x_i) & \text{if } x_j = In. \end{cases}$$

If $G(i)$ has been played in stage n, the leader in the next stage, i.e. stage $(n - 1)$, is

$$\text{firm } i \quad \text{if } x_j = Out$$
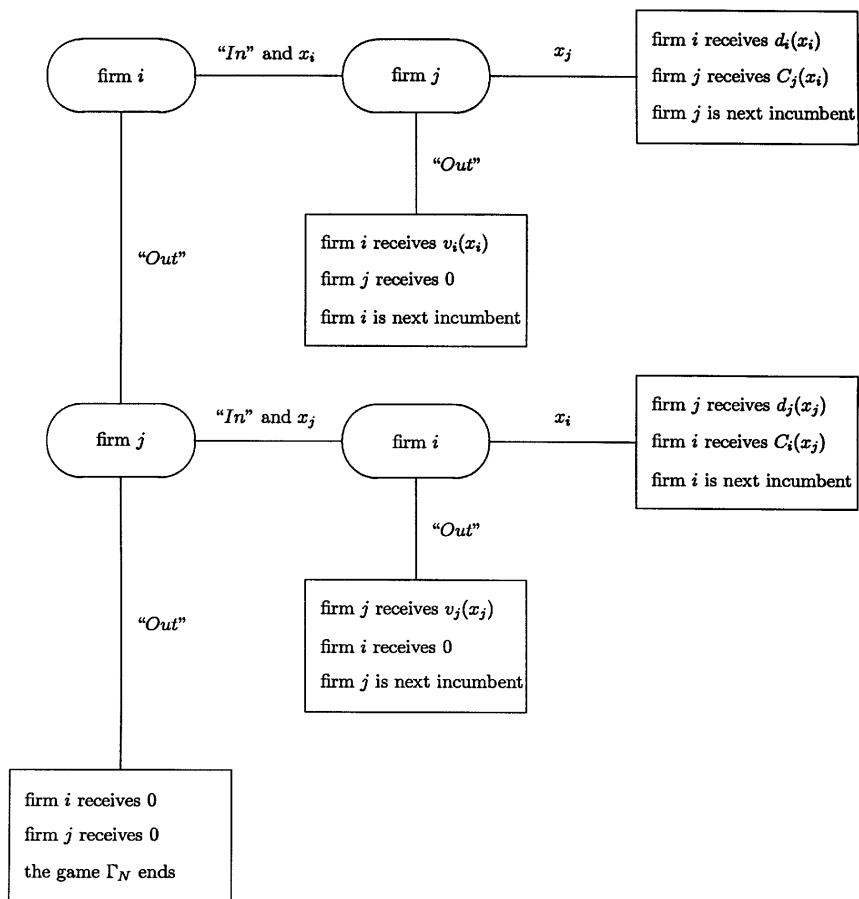$$\text{firm } j \quad \text{if } x_j = In.$$

FIGURE 1

Each firm's total payoff in the game is the sum of all its stage payoffs. We derive all our results in this no-discounting setting. They must be interpreted as limit results for low discounting.[1]

These rules capture in a reduced form the features of competition for leadership underlying most existing models of entry deterrence. To focus the discussion, let us compare it to Maskin and Tirole's framework. In static models, two attributes characterize the Stackelberg leader. First, when the leader moves, its rival has not committed to a policy. Second, when its rival moves, the leader has already committed to a policy. By analogy, a Stackelberg leader in Maskin and Tirole's rigid timing structure is a firm which either does not face any commitment by its rival at its decision date, or it has already imposed its own capacity commitment when the latter has to move. How does leadership change hands? Although the follower faces a commitment by the leader, it must invest aggressively when called to move, so as to deter its opponent from counterinvesting at its next decision date. Then, when the current follower will move again, it will not face any commitment and will have become de facto leader. In this process, both firms share the market for (at least) one period with low price and profits, as a result of the follower setting up a large capacity to deter future entry. The current leader certainly loses money in the overlapping period. The industry profits can be depressed enough so that the follower loses money over its initial two-period commitment. It takes an entry cost to acquire leadership and future incumbency. As for the current leader, should it forego an investment opportunity, it will move two periods later. However, at this time, it will have become the follower, i.e. if its rival has invested in the meantime.

Our model accounts for a similar leadership contest story, yet condenses it within one stage. The leader at the outset of the period, firm i, can commit to a policy x_i (e.g., price, quantity). If it operates as a monopoly, i.e. if firm j chooses not to counterinvest and plays "*Out*," firm i then earns a monopoly stage payoff $v_i(x_i)$. Otherwise, if firm j decides to challenge firm i's leadership and enters, firm i makes a nonpositive profit $d_i(x_i)$ in this stage. As to firm j, it faces firm i's short-term commitment x_i which affects its payoff in this leadership transfer phase; $C_j(x_i)$ is the *entry cost* for firm j against x_i, i.e. the losses that the follower has to incur to

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[1] In dynamic models, continuity at $\delta = 1$ of both the policies and the payoffs is generally problematic, as is the choice of an appropriate criterion in the undiscounted case (see Dutta, 1991, for a general theory). In our setting, the problem is much more trivial. We mostly deal with finite horizon problems, in which policies and payoffs can be shown to be continuous in the discount factor, including at $\delta = 1$, by a simple induction argument. We deal with infinite horizon only in the case of symmetric firms (Section 3.1) and explicitly state an equivalence result between the limit of the discounted case and the undiscounted case. The continuity result is directly provided by the key recursive equation.

enter. It is a reduced form for $\min_{x_j \in X(x_i)} \{C_j(x_i; x_j)\}$, where $X(x_i)$ is the set of actions that allow firm j to enter against x_i.

The following assumptions are made.

Assumption A. The functions v_1, v_2, C_1, and C_2 are continuous on X.

Assumption B. The functions v_1 and v_2 are strictly increasing on X.

Assumption C. The functions C_1 and C_2 are strictly decreasing on X.

Assumption D. The functions $(C_1 + v_1)$ and $(C_2 + v_2)$ are nonincreasing on X.

Assumption A involves no loss of generality. Assumptions B and C capture the idea that in order to build an entry barrier, the incumbent has to deviate from the policy of an unconstrained monopolist. Moreover, the higher the barrier, the greater the deviation and so the smaller the incumbent's profit. Consider, for instance, x_i as a price. The set X is the set of prices below the unconstrained monopoly price. Hence, the incumbent's profit is increasing in the price it charges. Conversely, since an entrant has to undercut the incumbent's price to enter, the lower this price, the lower the entrant's short-term profit, i.e. the larger the entry cost. Assumption D formalizes a very distinct property of our model; to raise its rival's entry cost by one unit, the incumbent has to reduce its own profit by less than one unit. In the subsequent analysis, this property is shown to be a sufficient condition for both the rent dissipation and the selection results.[2]

Note that only the functions "relative monotonicities" matter. In some contexts, such as quantity competition, it is innocuous but more elegant to assume the opposite monotonicities, i.e. v_i decreasing and C_i and $(C_i + v_i)$ increasing on X (see Section 4.2).

3. THE MAIN RESULTS

The analysis is in two steps. First, we study the case of symmetric firms which have the same functions v and C. Then, we examine a class of asymmetric entry games in which firms differ in their monopoly payoffs v_i and entry costs C_i.

3.1. *Symmetric Firms*

In the case of symmetric firms, $v_1 = v_2 = v$ and $C_1 = C_2 = C$. Define x^l by

$$v(x^l) = 0.$$

[2] However, as will be illustrated by some counterexamples, it is not a necessary condition.

The policy x^l allows the firm operating as a monopoly to just break even. If the strategic variables are prices, x^l is the monopolist's "average cost pricing" policy. Define x^L by

$$C(x^L) = 0.$$

The policy x^L is the largest $x \in X$ which, in the one-stage game, makes entry nonprofitable. Hence, by Assumption $\mathcal{B}$, x^L is the optimal entry-preventing policy in a one-stage game. In terms of prices, x^L is the "static limit pricing" policy.

In what follows, we assume that x^l and x^L exist and satisfy $x^l < x^L$. This ensures that in the only perfect equilibrium of $\Gamma_1(i)$, firm i plays "*In*" (i.e., $G(i)$ is played) and x^L, and then firm j plays "*Out*."[3]

In the N-stage game $\Gamma_N(i)$, entry is deterred by a dynamic entry-preventing policy that is a sequence of policies deterring entry in each stage of the game. The same firm maintains throughout the game. Its OEPP $\{x^N, x^{N-1}, \ldots, x^1\}$ is defined as the solution to

$$
\begin{aligned}
&x^1 = x^L \\
&C(x^{n+1}) = \textstyle\sum_{k=1}^n v(x^k) \quad \text{for } n = 1, \ldots, N-1.
\end{aligned}
\tag{1}
$$

This definition extends the notion of a static entry barrier to a dynamic framework. In each stage, a firm's OEPP sets its rival's entry cost equal to the total incumbency rent that the latter would earn as the incumbent in subsequent stages. This rent itself is determined by the OEPP that the new incumbent would have to follow. The OEPP is thus determined by dynamic programming. Observe that an equivalent recursive equation for the OEPP is

$$
\begin{aligned}
&x^1 = x^L \\
&C(x^{n+1}) = C(x^n) + v(x^n) \quad \text{for } n = 1, \ldots, N-1.
\end{aligned}
\tag{2}
$$

PROPOSITION 1.   *In the unique perfect equilibrium of any symmetric entry game $\Gamma_N(i)$, with $i \in \{1, 2\}$, firm i maintains with the OEPP $\{x^N, x^{N-1}, \ldots, x^1\}$.*

Proof.   We first prove by induction that the sequence $\{x^n\}_{n \in \mathbf{N}^*}$ is well defined, is decreasing, and satisfies $x^l \leq x^n \leq x^L$.

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[3]The assumption that x^l and x^L exist is only a matter of simplicity and is innocuous. We could use the more general definitions $x^l = \min\{x \mid v(x) \geq 0\}$ and $x^L = \max\{x \mid C(x) \leq 0\}$. The assumption that $x^l < x^L$ is common to all static entry games; i.e., absent this assumption, entry barriers are impossible.

Step 1. $x^1 = x^L > x^l$ by assumption. x^2 is defined by $C(x^2) = (C + v)(x^1)$. Then,

$$C(x^l) = (C + v)(x^l) \geq (C + v)(x^1) = C(x^2) \geq C(x^1) = C(x^L).$$

By the theorem of intermediate values, x^2 exists; it is unique by Assumption $\mathcal{C}$ and satisfies $x^l \leq x^2 \leq x^1 \leq x^L$.

Step $n + 1$. Suppose that until rank n, the OEPP exists, is unique, and satisfies

$$x^l \leq x^n \leq x^{n-1} \leq x^L.$$

Then

$$C(x^L) \leq C(x^n) = (C + v)(x^{n-1}) \leq (C + v)(x^n)$$
$$\leq (C + v)(x^l) = C(x^l)$$

which, given that $(C + v)(x^n) = C(x^{n+1})$, can be written

$$C(x^L) \leq C(x^n) \leq C(x^{n+1}) \leq C(x^l).$$

As in Step 1, x^{n+1} exists, is unique, and satisfies $x^l \leq x^{n+1} \leq x^n \leq x^L$.

We now prove by induction on N that $\Gamma_N(i)$ admits a unique perfect equilibrium in which firm i maintains as incumbent with the OEPP $\{x^N, x^{N-1}, \ldots, x^1\}$ and therefore earns a total rent $\sum_{n=1}^{N} v(x^n)$. The induction hypothesis is clearly satisfied for $N = 1$. Suppose that it holds until rank N and consider stage $(N + 1)$ in $\Gamma_{N+1}(i)$.

Suppose that firm i plays "*In*" and $x_i \leq x^{N+1}$. If firm j plays "*In*," its payoff is $-C(x_i) + \sum_{n=1}^{N} v(x^n) \leq 0$. Hence, firm j plays "*Out*" and firm i's payoff is then $v(x_i) + \sum_{n=1}^{N} v(x^n)$. Note that, for $x_i \leq x^{N+1}$, this payoff is maximized at x^{N+1} and that $v(x^{N+1}) > 0$.

Suppose that firm i plays "*In*" and $x_i > x^{N+1}$. Firm j enters because its total profit is then $-C(x_i) + \sum_{n=1}^{N} v(x^n) > 0$. Firm i's payoff is $d_i(x_i) < 0$.

Conditionally on playing "*In*," firm i's best strategy is to choose x^{N+1} and maintain.

Suppose that firm i plays "*Out*." Then $G(j)$ is played and, by the previous argument, firm j plays "*In*" and x^{N+1} and its OEPP. It earns $v(x^{N+1}) + \sum_{n=1}^{N} v(x^n) > 0$. It will thus, indeed, play "*In*" and firm i will earn 0.

Comparing the three options, firm i's optimal choice is "*In*" and x^{N+1}.
∎

COROLLARY 1. *As the number of remaining stages goes to infinity, the incumbent's stage payoff goes to zero, i.e.* $\lim_{n \to +\infty} x^n = x^l$.

Proof. Since the sequence $\{x^n\}_{n \in \mathbf{N}^*}$ is decreasing and bounded, it converges. From system (2), the limit x of $\{x^n\}_{n \in \mathbf{N}^*}$ has to satisfy $C(x) = C(x) + v(x)$, because C and $(C + v)$ are continuous. Hence, $v(x) = 0$, i.e., $x = x^l$. ■

Corollary 1 shows that our finite horizon model is consistent with the infinite horizon models found in the literature. More precisely, the infinite horizon models (Eaton and Lipsey, 1980; Maskin and Tirole, 1988) generate a stationary OEPP for each value of the discounting factor. As the discounting factor goes to 1 (i.e., when cash flows are less discounted), the OEPP converges towards the policy such that stage payoffs equal zero. This constitutes the celebrated *rent dissipation property*. In our approach, in which the discounting factor is directly assumed to equal 1, the same policy obtains as the limit when the horizon goes to infinity.[4] The flow profit and therefore the average rent converge to zero. Note, however, that the total rent does not.

COROLLARY 2. *The incumbent's rent converges to* $C(x^l)$. *That is,*

$$\lim_{N \to \infty} \sum_{n=1}^{N} v(x^n) = C(x^l) > 0.$$

Proof. The incumbent's rent in $\Gamma_N(i)$ is $\sum_{n=1}^{N} v(x^n) = C(x^{N+1})$ which converges to $C(x^l)$ as N goes to infinity. $x^l < x^L$ implies that $C(x^l) > 0$. ■

Notice that in some models the function v may not satisfy Assumption $\mathcal{B}$ on the entire range of values of x (see Section 4.2 for an example). Moreover, it may be that x^M, the optimal policy of an unconstrained monopoly, actually deters entry in the one-stage game. That is, the threat of entry does not constrain the monopolist's policy choice in the one-stage game. The incumbent's OEPP may be stationary at x^M for short horizon games, but policies are constrained for longer horizons and the same monotonic convergence result is obtained when the horizon goes to infinity.

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[4] The convergence towards the solution of infinite horizon models is not due to the no-discounting assumption. It can be shown that for any discounting factor, the OEPP obtained in our finite horizon model converges to the stationary OEPP obtained in infinite horizon models. Indeed, in our approach, the limit of the OEPP has to solve a recursive equation which is precisely the fundamental recursive equation common to all infinite horizon models.

3.2. *Asymmetric firms*

Consider now the case of two firms with respective efficiency levels S and W. Relative efficiency is captured by the following assumption (pertaining to functions).

Assumption E. $v_W < v_S$ and $C_W \geq C_S$.

Assumption E states that for a given policy, the stage payoff of a strong firm is strictly greater than that of a weak firm. Furthermore, the entry cost against a given policy is smaller for a strong than for a weak firm.

Define (x_W^l, x_S^l) and (x_W^L, x_S^L) by

$$v_W(x_W^l) = 0, \quad v_S(x_S^l) = 0,$$
$$C_W(x_S^L) = 0, \quad C_S(x_W^L) = 0.$$

Assumption E implies

$$x_S^l < x_W^l.$$

Indeed, the strong firm makes positive profits for policies that do not allow the weak one to break even. Moreover,

$$x_W^L \leq x_S^L.$$

That is, preventing entry by the more efficient firm in the one-stage game requires a (weakly) more "aggressive" policy than preventing entry by the less efficient firm. Assuming $x_W^l < x_W^L$, the weak firm can profitably prevent entry in $\Gamma_1(W)$ with x_W^L. We prove, however, that the weak firm is unable to maintain indefinitely in $\Gamma_N(W)$, with N large enough.

When firms are unequally efficient, their OEPPs differ in each stage. The system which extends system (1) derived in the symmetric case is[5]

$$
\begin{aligned}
x_W^1 &= x_W^L \\
x_S^1 &= x_S^L \\
C_W(x_S^{n+1}) &= \sum_{k=1}^n v_W(x_W^k) \quad \text{for } n = 1, \ldots, N-1 \\
C_S(x_W^{n+1}) &= \sum_{k=1}^n v_S(x_S^k) \quad \text{for } n = 1, \ldots, N-1.
\end{aligned}
\tag{3}
$$

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[5]This system is equivalent to the recursive system

$$
\begin{aligned}
x_W^1 &= x_W^L \\
x_S^1 &= x_S^L \\
C_W(x_S^{n+1}) &= C_W(x_S^n) + v_W(x_W^n) \quad \text{for } n = 1, \ldots, N-1 \\
C_S(x_W^{n+1}) &= C_S(x_W^n) + v_S(x_S^n) \quad \text{for } n = 1, \ldots, N-1.
\end{aligned}
$$

It is expected that, when defined, $\{x_S^n\}_{n \leq N}$ and $\{x_W^n\}_{n \leq N}$ correspond to the respective OEPPs of the strong and the weak firms in $\Gamma_N(S)$ and $\Gamma_N(W)$. Indeed, it easy to show, as in Proposition 1, that if $\sum_{n=1}^{N} v_W(x_W^n) \geq 0$ then $\sum_{n=1}^{N} v_S(x_S^n) \geq 0$ and for $i \in \{W,S\}$, $\Gamma_N(i)$ has a unique perfect equilibrium in which firm i maintains using the OEPP $\{x_i^N, \ldots, x_i^1\}$. However, this does not hold for N large as we explain below.

LEMMA 1. *Let* $\{y_W^n\}_{n \in \mathbf{N}^*}$ *be the OEPP in the symmetric case with two weak firms. For all* n *for which* x_W^n *and* x_S^n *are defined, the following inequalities hold*: $x_W^n \leq y_W^n \leq x_S^n$.

Proof. By induction.

Step 1. By definition, $x_W^1 = x_W^L \leq x_S^L = y_W^1 = x_S^1$.

Step $n + 1$. Suppose $x_W^k \leq y_W^k \leq x_S^k$, $\forall k \leq n$:

$$C_W\left(x_S^{n+1}\right) = \sum_{k=1}^{n} v_W\left(x_W^k\right) \leq \sum_{k=1}^{n} v_W\left(y_W^k\right) = C_W\left(y_W^{n+1}\right).$$

Hence, $x_S^{n+1} \geq y_W^{n+1}$. Moreover,

$$C_S\left(x_W^{n+1}\right) = \sum_{k=1}^{n} v_S\left(x_S^k\right) \geq \sum_{k=1}^{n} v_S\left(y_W^k\right) > \sum_{k=1}^{n} v_W\left(y_W^k\right)$$
$$> C_W\left(y_W^{n+1}\right) \geq C_S\left(y_W^{n+1}\right)$$

which implies $x_W^{n+1} < y_W^{n+1}$. ∎

LEMMA 2. $\exists M \in \mathbf{N}^*$ *such that*

(i) *either* $\sum_{k=1}^{n} v_W(x_W^k) < 0$

(ii) *or* x_W^n *is not defined*

if and only if $n \geq M$.

Proof. First, note that if $\{x_W^n, \ldots, x_W^1\}$ is defined and $\sum_{k=1}^{n} v_W(x_W^k) \geq 0$ then x_S^{n+1} is defined. Indeed, $C_W(x_S^L) = 0 \leq \sum_{k=1}^{n} v_W(x_W^k) \leq \sum_{k=1}^{n} v_W(y_W^k) = C_W(y_W^n)$ and so x_S^{n+1}, defined by $C_W(x_S^{n+1}) = \sum_{k=1}^{n} v_W(x_W^k)$, exists by the theorem of intermediate values.

Let M_0 be the first rank (possibly infinite) at which x_W^n is not defined, i.e., $M_0 = \max\{n \mid \sum_{k=1}^{n-1} v_S(x_S^k) \leq \max_{x_W \in X} C_S(x_W)\}$.

(i) First, consider the case $M_0 = +\infty$. Let us define $M < M_0$ by $M = \min\{n \mid \sum_{k=1}^{n} v_W(x_W^k) < 0\}$ and show that M is finite. By Lemma 1, $\sum_{k=1}^{n} v_S(x_S^k) \geq \sum_{k=1}^{n} v_S(y_W^k) \geq n \cdot v_S(x_W^l)$. Since $v_S(x_W^l) > v_W(x_W^l) = 0$, there exists n_0 such that $\sum_{k=1}^{n_0} v_S(x_S^k) > C_S(x_W^l)$. Hence, for $n > n_0$,

$x_W^{n+1} < x_W^n < x_W^l$. Hence, for $n > n_0$, $\sum_{k=1}^n v_W(x_W^k) < \sum_{k=1}^{n_0} v_W(x_W^k) + (n - n_0) \cdot v_W(v_W^{n_0+1})$ which decreases towards $-\infty$.

(ii) Consider now the case $M_0 < +\infty$.

If $\sum_{k=1}^{M_0-1} v_W(x_W^k) \geq 0$ then $M = M_0$.

If $\sum_{k=1}^{M_0} v_W(x_W^k) < 0$, we can define $M < M_0$ by $M = \min\{n \mid \sum_{k=1}^n v_W(x_W^k) < 0\}$. ∎

If the horizon is short enough, the weak firm can maintain and make positive profits in all stages. For longer horizons, in order to maintain, the weak firm has to incur negative profits in early stages, which are compensated by the positive profits in later stages. Lemma 2 states that for horizons long enough, one of two circumstances arises first: (i) either the weak firm is *able but unwilling* to deter entry because the losses that it would have to incur outweigh the positive rent it would eventually earn, or (ii) the strong firm's incumbency rent is so high that the weak firm is *unable* to deter entry. In such games, despite being the initial leader, the weak firm is better off exiting the market. The strong firm enters and maintains in the long run. This leads directly to the selection property, which is the paper's central result.

PROPOSITION 2.   *In the unique perfect equilibrium of any asymmetric entry game $\Gamma_N(W)$ with $N \geq M$, the weak firm exits and remains out after at most $(M - 1)$ stages.*

Proof. The weak firm can profitably maintain as the incumbent in $\Gamma_{M-1}(W)$ but not in $\Gamma_M(W)$. In $\Gamma_M(W)$, its first move is "*Out.*" Then, $\Gamma_M(W)$ is played as $\Gamma_M(S)$ and the strong firm maintains with the OEPP $\{x_S^M, \ldots, x_S^1\}$.

Consider now a game of length $N > M$ and let $N = Q \cdot M + R$, with $R < M$. In terms of its subgame perfect equilibria, the game $\Gamma_N(i)$ can be analyzed as an introductory game $\Gamma_R(i)$ followed by Q independent games $\Gamma_M(S)$. Indeed, since the termination subgame is in any case played as $\Gamma_M(S)$, $\Gamma_{2M}(i)$ is actually played as $\Gamma_M(i)$, followed by $\Gamma_M(S)$. By the same argument used for the M-period endgame, one shows that $\Gamma_M(i)$, the first M-period game of $\Gamma_{2M}(i)$ is played as $\Gamma_M(S)$ independently of i. $\Gamma_{2M}(i)$ is therefore played as $\Gamma_{2M}(S)$, that is, two independent $\Gamma_M(S)$ in a row. By the same token, one gets that $\Gamma_N(i)$ is played as $\Gamma_R(i)$, followed by Q independent $\Gamma_M(S)$. ∎

Some comments are in order. First, this solution naturally extends that of static entry games. If the asymmetry is very large, $x_W^l > x_W^L$ and the weak firm cannot deter entry even in a one-stage game. $\Gamma_N(W)$ and $\Gamma_N(S)$ have the same perfect equilibrium outcome; the strong firm is in the market and follows its static entry-preventing policy x_S^L in each stage. That

is, an N-stage game is played as a sequence of independent one-shot entry games. With smaller asymmetries, the weak incumbent can maintain in short games. However, it eventually has to give up incumbency in longer games, except for an introductory phase (of less than M stages). Once the weak firm has exited, such games are played as a sequence of independent games $\Gamma_M(S)$, in each of which the strong firm maintains using its OEPP $\{x_S^M, \ldots, x_S^1\}$. That is, once the strong firm is in the market, it follows a cyclical OEPP. By backward induction, this complete information game is played as a sequence of independent games. During the R first stages, the weak incumbent deters entry just as in $\Gamma_R(W)$, anticipating that in stage $Q \cdot M$ the strong firm will enter. Conversely, the strong firm is certain to enter in stage $Q \cdot M$. Its prize for entering earlier would thus merely be incumbency in some of the R first stages. By definition of the weak firm's OEPP, the prize is not worth the entry cost.[6]

Second, the firms' average equilibrium profits reflect their relative efficiency. Since the weak firm cannot maintain in more than a given (finite) number of stages, its average profit converges to zero when the horizon goes to infinity. Instead, the strong firm's average profit converges to $(1/M) \sum_{k=1}^{M} v_S(x_S^k) > 0$.

Finally, as the efficiency differential goes to zero,[7] the solution of the asymmetric case converges to that of the symmetric case. The convergence holds pointwise for any term of the sequences $\{x_W^n\}_{n \in \mathbf{N}^*}$, $\{x_S^n\}_{n \in \mathbf{N}^*}$: $\lim_{W \to S} x_W^n = \lim_{W \to S} x_S^n = y_S^n$, by continuity of the solutions to system (3). It is then straightforward to establish by contradiction that $M = \min\{n \in \mathbf{N} : \sum_{k=1}^{n} v_W(x_W^k) < 0\} \to +\infty$ when $W \to S$. In the limit, the weak firm has become as efficient as the strong one and can maintain over games of arbitrarily long horizons.

3.3. An Extension with Infinite Horizon

The analysis of competition between asymmetric firms cannot be extended directly to an infinite horizon setting. In the finite horizon game, the strong firm's OEPP is cyclical; such a policy does not converge when the length of the game goes to infinity. Yet, in many situations of economic relevance, efficiency asymmetries between firms are only temporary. This is the case, for example, when the efficiency advantage comes from a patented innovation; when the patent expires, all firms in the industry can use the most efficient technology. Such situations can be analyzed through a simple extension of our basic model. Moreover, a similar issue of

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[6] Note that this would not be the case with multiple strong entrants because these would enter as early as possible so as to preempt the market.

[7] Formally, the functions v_W, C_W, and d_W converge uniformly to the functions v_S, C_S and d_S on X.

selection can be raised: to what extent can an inefficient incumbent maintain in the expectation of catching up later?

Consider the following infinite horizon game. In the first N stages (stages N, $N - 1, \ldots, 1$), the firms are asymmetric with respective efficiencies S and W. This phase is followed by an infinite horizon endgame (stages 0, -1, $-2, \ldots$) in which both firms have the same efficiency level S. The whole game is denoted $\Gamma_N^\infty(i)$, where firm i is the incumbent in stage N. In the symmetric endgame, the incumbent maintains indefinitely with its limit OEPP x_S^l and earns a cumulative incumbency rent $C_S(x_S^l)$ (see Corollaries 1 and 2). Let $\{x_W^N, \ldots, x_W^1\}$ and $\{x_S^N, \ldots, x_S^1\}$ be the respective OEPPs of the weak and the strong firms during the asymmetric phase of $\Gamma_N^\infty(W)$ and $\Gamma_N^\infty(S)$. These are determined along the lines exposed in 3.2, except that the incumbent in the symmetric endgame earns a rent $C_S(x_S^l)$. Accordingly, the system extending system (3) is

$$
\begin{aligned}
C_W(x_W^1) &= C_S(x_S^l) \\
C_S(x_S^1) &= C_S(x_S^l) \\
C_W(x_S^{n+1}) &= \sum_{k=1}^{n} v_W(x_W^k) + C_S(x_S^l) \quad \text{for } n = 1, \ldots, N-1 \\
C_S(x_W^{n+1}) &= \sum_{k=1}^{n} v_S(x_S^k) + C_S(x_S^l) \quad \text{for } n = 1, \ldots, N-1.
\end{aligned}
\tag{4}
$$

LEMMA 3. *For all $n \geq 0$ for which x_W^n and x_S^n are defined, the following inequalities hold*: $x_W^n \leq x_S^l \leq x_S^n$.

Proof. By induction.

Step 0. $x_W^0 = x_S^0 = x_S^l$ by assumption.

Step $n + 1$. Suppose that the inequalities hold until rank n:

$$
\begin{aligned}
C_W(x_S^{n+1}) &= \sum_{k=1}^{n} v_W(x_W^k) + C_S(x_S^l) \\
&< \sum_{k=1}^{n} v_S(x_W^k) + C_S(x_S^l) \\
&< C_S(x_S^l) \leq C_W(x_S^l),
\end{aligned}
$$

where we used respectively $v_W < v_S$, the inequalities hold until rank n, which ensure that each term $v_S(x_W^k)$ is negative, and $C_S \leq C_W$. It follows that $x_S^{n+1} \geq x_S^l$. Similarly,

$$
C_S(x_W^{n+1}) = \sum_{k=1}^{n} v_S(x_S^k) + C_S(x_S^l) \geq C_S(x_S^l).
$$

Hence $x_W^{n+1} \leq x_S^l$. ∎

PROPOSITION 3.   *There exists a maximal number of stages N such that, in the unique perfect equilibrium of $\Gamma_{N-1}^{\infty}(W)$, the weak firm maintains permanently.*

Proof.   Lemma 3 implies that during the asymmetric phase, in each stage in which the weak firm is active, its stage payoff is strictly negative: $v_W(x_W^k) \leq v_W(x_S^l) < v_S(x_S^l) = 0$. As in Lemma 2, there exists a smallest N such that either its OEPP is undefined or its total incumbency rent in $\Gamma_N^{\infty}(W)$ earned with its OEPP $\{x_W^N, \ldots, x_W^1\}$ becomes strictly negative: $\sum_{k=1}^{N} v_W(x_W^k) + C_S(x_S^l) < 0 \leq \sum_{k=1}^{N-1} v_W(x_W^k) + C_S(x_S^l)$. As in Proposition 2, only the strong firm can be incumbent in $\Gamma_N^{\infty}(i)$, independently of whether $i = W$ or S.   ∎

Besides its providing a framework to handle some infinite horizon problems, the analysis of Γ_N^{∞} sheds light on the role of Assumptions $\mathcal{A}$ to $\mathcal{D}$.

First, the different approaches quoted in this paper predict that in symmetric infinite horizon situations the same firm maintains "forever" and the rent dissipation property holds, with or without Assumption $\mathcal{D}$. Second, Assumption $\mathcal{D}$ ensures that in Γ_N, the sequence $\{x^n\}_{n \in \mathbf{N}^*}$ converges towards x^l. As a consequence of Assumptions $\mathcal{E}$, one establishes the existence of an n such that $v_W(x_W^n) < 0 < v_S(x_S^n)$. Considering system (3), it is straightforward to prove by induction that for $n' > n$, when $x_W^{n'}$ is defined, $v_W(x_W^{n'}) < v_W(x_W^n) < 0 < v_S(x_S^n) < v_S(x_S^{n'})$. That is, consider a game of given horizon and suppose that the weak firm must lose money in the early stages to maintain in later ones. Then, in a game with a longer horizon, the weak firm must lose strictly more money to remain the incumbent.

In $\Gamma_N^{\infty}(W)$, the selection result derives from the same mechanism, except that the weak incumbent would lose money even within a single period of asymmetry. In $\Gamma_1^{\infty}(W)$, the weak incumbent has to deter the entry of a strong firm aiming at an incumbency rent $C_S(x_S^l)$ earned in the symmetric endgame. This means charging x_S^l which is below the weak incumbent's average cost policy. It could be worth making a bounded loss so as to be the incumbent when the symmetric endgame starts and earn a rent $C_S(x_S^l)$. Yet, with long phases of asymmetry, such losses would accumulate and eventually outweigh the rent, making entry prevention unprofitable or impossible for the weak firm. Here, Assumption $\mathcal{D}$ is not necessary to find the existence of a minimal duration of asymmetry such that the weak incumbent is forced to lose money to preserve incumbency.[8]

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[8] Observe that N and M as determined in Proposition 2 have no obvious connections; they are derived from identical recursive systems but with different starting points.

4. THREE EXAMPLES

This section illustrates how v and C are constructed in specific economic contexts. It emphasizes the selection result in three models—price competition with almost homogeneous products, quantity competition, and the renewal of productive capacities—which have been previously analyzed in the rent dissipation perspective only. In each of them, firms differ in their fixed costs. Assumption E is straightforward to check. Assumption D, which is central to our analysis of the rent dissipation and selection results, is given particular attention.

4.1. *Limit Pricing*

This first example is a direct adaptation of Ponssard (1991) to a sequential move setting.[9] The stage games are based on a model of price competition with almost homogenous products. Let p_i denote firm i's price. When p_1 and p_2 are close enough and not too high, both firms are active and the duopoly demand function for firm i is

$$d_i^d(p_i, p_j) = (1 + \omega)(1 - p_i + \omega(p_j - p_i))/(1 + 2\omega) \quad \text{with} \quad i \neq j.$$

If by contrast the price differential is large, the firm with the lowest price has a monopoly demand:

$$d_i^m(p_i) = 1 - p_i.$$

These definitions generate a piecewise continuous, kinked demand curve. With ω large enough, the demand goes entirely to the low price firm, except for very small price differences. The natural monopoly structure of the market pertains to the existence of fixed costs, F_1 and F_2, incurred in case of production. The monopoly flow profit is

$$v_i(p_i) = (1 - p_i)p_i - F_i.$$

This is an increasing function of $p_i \in [0, \frac{1}{2}]$.

With large enough fixed costs and a large enough ω, the Stackelberg one-stage game has the first mover (the incumbent) cornering the market with a limit-pricing policy. Leadership and incumbency refer here to an idea of consumers' switching costs. To steal the consumers from the current incumbent and become leader in the next period, the entrant has to undercut its rival by a strict margin depending on the price charged by

the incumbent. Precisely, the price p_j minimizing firm j's entry cost against p_i is

$$p_j(p_i) = ((1 + \omega)p_i - 1)/\omega,$$

and the corresponding entry cost is

$$C_j(p_i) = F_j - (1 + \omega)(1 - p_i)((1 + \omega)p_i - 1)/\omega^2.$$

For ω large enough, C_j is decreasing on $[0, \frac{1}{2}]$. Moreover, Assumption $\mathcal{E}$ holds. In the case of symmetric firms, we have

$$(v + C)(p) = (1 + 2\omega)(1 - p)((1 + \omega)/(1 + 2\omega) - p)/\omega^2,$$

$(v + C)$ is decreasing, and Assumption $\mathcal{D}$ holds. With ω large enough, the rent dissipation and selection properties are therefore obtained.[10]

4.2. *Capacity as an Entry Barrier*

Quantity competition is a most natural setting to investigate issues of entry-deterrence. Interpreted as a capacity, a quantity has the value of a credible commitment to an entry-preventing behavior (Dixit, 1979). In the simplest version, firms compete à la Cournot, with a linear demand. Firm i's profit (stage payoffs) is a function of its capacity decision, k_i, as well as its rival's, k_j:

$$v_i(k_i, k_j) \begin{cases} = k_i(1 - k_i - k_j) - F_i & \text{if } k_i > 0 \\ = 0 & \text{if } k_i = 0. \end{cases}$$

The market is assumed to have a natural monopoly structure due to large fixed costs of production, F_1 and F_2. As a firm's revenue is necessarily smaller than the unconstrained monopoly profit $\frac{1}{4}$ (corresponding to a quantity $\frac{1}{2}$) we assume that $\max(F_1, F_2) < \frac{1}{4} < (F_1 + F_2)$ so that, in equilibrium, only one firm produces in each stage.

Maskin and Tirole analyze an infinite horizon, dynamic version of a similar game with symmetric firms. In their model, the two firms move in alternance and their decisions commit them for two periods. Investigating the Markov perfect equilibria of this game, in which each firm reacts only to its rival's last move, they prove that the rent dissipation property obtains: the equilibrium strategies are trigger strategies (the firm producing nothing or a large quantity depending on its rival's last move) and the

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[10]ω large (with respect to the fixed costs) is needed to ensure that the two firms cannot profitably share the market. Also, with ω small, each firm has a limited market power. As a result, the incumbent might be better off accommodating rather than deterring entry.

trigger quantity converges to the average cost policy as discounting decreases.

However, the same Markov equilibrium approach used for asymmetric firms does not lead to the selection property (Lahmandi-Ayed *et al.*, 1996). Instead, the less efficient firm maintains indefinitely and, furthermore, earns the same total incumbency rent as the most efficient firm would; while its production cost is greater, its equilibrium trigger quantity is smaller than that of the efficient firm. This result is counterintuitive because the less efficient firm's OEPP is *less* constrained than that of the more efficient one. It is thus worthwhile to consider a finite horizon model, as discussed in Section 2. Are the rent dissipation and selection properties obtained naturally?

Unlike for price competition, entry deterrence consists in *inflating* capacities. Hence, we should check that v, C, and $(v + C)$ have monotonicities opposite to those in Assumptions $\mathcal{B}$, $\mathcal{C}$, and $\mathcal{D}$.[11] The monopoly stage payoff can be written

$$v_i^m(k_i) = k_i(1 - k_i) - F_i,$$

where v_i is decreasing for $k_i \in [\frac{1}{2}, 1]$, which is the relevant range because the incumbent will deter entry by expanding its capacity beyond its unconstrained monopoly level, $\frac{1}{2}$. Given a capacity $k_i \geq \frac{1}{2}$ for firm i, firm j minimizes its entry cost by maximizing its short-run revenue and choosing $k_j(k_i) = (1 - k_i)/2$. Its entry cost is then

$$C_j(k_i) = -v_j\left(\frac{1 - k_i}{2}, k_i\right) = F_j - (1 - k_i)^2/4.$$

Hence C_j is increasing for $k_i \in [\frac{1}{2}, 1]$. If firm j enters, firm i earns

$$d_i(k_i) = v_i\left(k_i, \frac{1 - k_i}{2}\right) - F_i = k_i(1 - k_i)/2 - F_i.$$

In the case of symmetric firms, the function $(v + C)$ can be written

$$(v + C)(k) = k(1 - k) - (1 - k)^2/4 = (1 - k)(5k - 1)/4,$$

where $(v + C)$ is increasing for $k \leq \frac{3}{5}$. For the relevant range for k to be $(\frac{1}{2}, \frac{3}{5})$, the fixed cost must be in $(0.24, 0.25)$. Indeed, it must be greater than 0.24 so that $\frac{3}{5}$ is an upper bound to capacities allowing a firm to break even, while 0.25 is the maximum revenue that an incumbent can earn that

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[11] Equivalently, one could consider $x_i = -k_i$ as the strategic variable and check the monotonicities as in Assumptions $\mathcal{B}$, $\mathcal{C}$, and $\mathcal{D}$.

must cover its fixed cost. As long as the fixed cost is in (0.24, 0.25), the rent dissipation and selection properties naturally hold. However, as soon as the fixed cost is below 0.24, even the rent dissipation property does not obtain in this model.

It is instructive to contrast the price and the quantity models in terms of their strategic variables and the corresponding reaction functions for an entrant. The first example deals with strategic complements for which the entry price amplifies the incumbent's price; to a low price by the incumbent corresponds an even lower entry price. The entry cost varies widely with the incumbent's price. By contrast, the second example deals with strategic substitutes; the optimal policy to enter is not very sensitive to the incumbent's policy $(\partial k_j(k_i)/\partial k_i = -\frac{1}{2})$. A change in the incumbent's policy affects only mildly the entry cost, contrary to what Assumption $\mathcal{D}$ requires. Therefore, the rent dissipation and selection properties obtain more naturally with strategic complements than with strategic substitutes.[12]

4.3. *Durable capital as an entry barrier*

The last example is adapted from the plant renewal model of Eaton and Lipsey (1980). Time is taken to be continuous, to facilitate the calculations. Since there are well-known difficulties with subgame perfection in continuous time games (see Simon and Stinchcombe, 1989), all our results should be read as limit results for discrete time games, when the grid of moves becomes arbitrarily fine. Production requires setting up a plant that becomes obsolete after H units of time. To set it up, firm i, with $i \in \{1, 2\}$, incurs a large fixed cost F_i. Operating costs are assumed to be zero. If both firms are in the market, their duopoly flow of revenue is π_d per unit of time. If only one of them is present, its flow of revenue is π_m.

Two additional assumptions are made. First, $\max(F_1, F_2) < \pi_m H < (F_1 + F_2)$, so that a single firm operates in equilibrium. Second, $\pi_d < \pi_m/2$; i.e., duopoly competition depresses profits.

Eaton and Lipsey study entry-prevention through premature renewal of capital. The strategic variables are the dates at which the firms set up new plants. They characterize the incumbent's OEPP in the case of symmetric firms and show that the corresponding premature renewal of plants leads to rent dissipation when the discounting is low enough. In the limit, the incumbent renews its plant as soon as it has covered its fixed cost of installation. Their rational expectations argument, which leads to the

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[12] Lahmandi-Ayed (1995) considers a model in which the entrant becomes the next leader only if it sets a capacity higher than that of the incumbent. This "escalation rule" restores the desired results. Indeed, it aligns the entrant's decision on the incumbent's $(k_j(k_i) = k_i)$ and ensures that the entry cost varies as much as the monopoly profit does when the incumbent increases its capacity.

classical recursive formulation, cannot be extended as such to the case of asymmetric firms.

In order to examine the selection issue, we consider instead the finite horizon version of their capital replacement game. By a finite horizon N it is meant that only a fixed, finite number N of equipment can be set up in the industry as a whole and that firms compete for the installment of these plants.[13] For $N = 2$, the first equipment can be replaced only once, either by the incumbent or by its rival. In particular the last equipment will be exploited until the end of its natural lifetime. As opposed to the two previous models, no other assumption than the fixed number of plants is needed to directly set the game as a sequential move game with endogenous leadership which can be solved within our framework.

Suppose that a firm renews its plant at date t_i. The new plant duplicates the existing capacities during $(H - t_i)$ time units but extends the production lifetime by t_i time units. The monopoly stage payoff associated with this new plant is then defined as its *additional value* with respect to the existing capacities set up by the same firm,

$$v_i(t_i) = \pi_m t_i - F_i.$$

It is increasing in t_i. Given a planned renewal date t_i, the entry-cost-minimizing strategy for firm j consists in preempting the next plant, i.e. to set it up immediately prior to t_i. Since the previous incumbent's equipment still has $H - t_i$ units of time to go, firm j has to share the market with firm i during these $H - t_i$ units and earns the duopoly flow of revenue π_d. The entry cost is defined as the loss in revenue due to the initial duopoly phase:

$$C_j(t_i) = (\pi_m - \pi_d)(H - t_i).$$

C_j is decreasing in t_i.

Considering symmetric firms, we have $(v + C)(t) = \pi_d \cdot t + \alpha$, where α is a constant. Then, $(v + C)$ is decreasing in t if and only if $\pi_d \leq 0$; rent dissipation and selection are obtained by assuming that duopoly destroys revenues.

The phenomena underlying the rent dissipation and selection results are best captured in the case $\pi_d = 0$. Entry barriers coincide with premature renewal of capital, provided that, once the plant is set up, the incumbent is committed to be in the market until its obsolescence. Then, renewing

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[13] Interpreting a finite horizon as a situation in which there is a fixed, finite number of plants to be set up simplifies the analysis. An alternative would be to consider a limited time horizon, which would lead to the same type of results, although at the cost of a considerable analytical and notational burden.

capital prematurely implies that the entrant always faces a minimal residual phase of duopoly competition if it enters, which partly destroys its revenues. The entry-preventing policy forces the incumbent to reduce the exploitation period of each plant. With symmetric firms and $\pi_d = 0$, the rent dissipation is readily obtained since the incumbent does not earn any profit on each new plant. Indeed, the function $(C + v)$ is constant and the sequence of renewal dates is stationary:

$$t^1 = \cdots = t^n = \cdots = F/\pi_m.$$

With asymmetric firms, Lemma 2 shows that the less efficient firm has to renew at an even more rapid pace in order to maintain. It then loses (more than a fixed amount of) money on each new plant. Incumbency can be profitable to the less efficient firm only if the profit earned over the first plant outweighs the additional loss incurred for all subsequent plants. Obviously, for a large enough number of plant renewals, this cannot be the case and the less efficient firm is better off exiting; the more efficient firm is selected as the long-run incumbent.

Moreover, Steinmetz (1996) shows that the rent dissipation and selection results might hold as well for positive values of π_d, i.e. in the case of a less destructive duopoly competition. This illustrates in particular that Assumption $\mathcal{D}$ is a sufficient, but not a necessary, condition.

5. CONCLUSION

In dynamic entry-deterrence contexts, whenever firms are asymmetric, *selection* is the very first economic property to investigate. This article provides sufficient conditions for this property to hold, using a simple game theoretic framework. It also shows that these conditions lead quite naturally to the already investigated *rent dissipation property* in the special case of symmetric firms. Thus this article constitutes an extension of the existing literature on entry. Three specific examples are examined which illustrate the generality of our framework and, in particular, the context in which our sufficient conditions can arise naturally. They also point out that these conditions are not necessary ones.

In order to focus on the selection question, several issues have been left aside. The main short-coming of our approach may be the assumption of a finite horizon. While the selection property arises very naturally in such a setting, some equilibrium features are not very appealing, especially the cyclicity of the strong firm's OEPP. Hence a first potential avenue of future research would be to formulate a solution for general infinite horizon games which captures our effects. One can hope, in particular, to

reconcile the Markov equilibrium approach with ours. Another line of future research is to extend the analysis to more general situations. For instance, the results should carry through to natural oligopolies. The process of leadership transfer has been modelled in a reduced form. It could be developed and refined, e.g., to describe the incumbent's exit decision. Finally, new interesting effects, such as reputation building, arise when firms have private information about their level of efficiency. The formal analysis of these questions is left for future research.

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