JEAN-PIERRE PONSSARD

SELF ENFORCEABLE PATHS IN EXTENSIVE FORM GAMES

A Behavioral Approach Based on Interactivity

ABSTRACT. This paper explores the idea of forward induction for extensive games. It interprets this idea as a general behavioral principle the technical details of which have to be worked out in each specific case. Because of its cooperative ingredient, this approach should be contrasted with the usual approaches of non-cooperative game theory which are rooted in individual rationality.

Keywords: Forward induction, Nash refinements, focal points.

1. INTRODUCTION

For a long time it had been accepted that a game in extensive form contained more information than its normal form counterpart. Accordingly solution concepts should be adapted to this specificity. Perfect equilibrium is one such attempt in this direction (Selten, 1975). But in the recent years this view has been under attack in particular with the possible refinements of Nash equilibria derived directly from the normal form (Kohlberg and Mertens, 1986). It is a fact that this normal form approach sometimes generates much more meaningful solutions than an extensive form analysis would. Nevertheless this paper seeks to restore the initial standpoint.

The idea is to define self enforceability directly in the extensive form, but not self enforceability of an equilibrium, self enforceability of a path in the game tree. Then threats are irrelevant, the only thing that matters is that no unilateral deviation may induce a more preferable outcome for the deviating player. As a matter of fact this idea, known as 'forward induction', is related to the normal form approach of Kohlberg and Mertens, but only indirectly (Van Damme, 1987).

This approach is very different from the perfect equilibrium approach in the sense that it involves some tacit cooperation which requires that errors will never be played and that only self enforceable

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paths are looked for. It may be interpreted as a form of preplay communication the goal of which is to design general principles to solve a class of games. Then a specific path is proposed for a specific game but the players are constantly encouraged to achieve a superior payoff through a non-ambiguous deviation as long as it remains consistent with the shared principles. The details of these shared principles can only be known when the singularities of the game to be played are exactly known. Indeed it is the feeling that these principles seem impossible to be fully operationalized generically for all extensive games that makes this whole approach meaningful, keeping a real potential for decentralized action. As such this approach of self enforceability may be viewed as a weakening of the assumption that the players have common knowledge about the game and the solution concept to be used. This assumption is critical to many results (Arrow, 1986).

This view of extensive games seems appropriate to study economic situations. Contrarily to parlor games, economic situations are never closed. The players are expected to innovate and the very notion of rules of the games is elusive. Then, in spite of its preliminary and abstract state, this standpoint is encouraging for many applications in which common knowledge is inadequate.

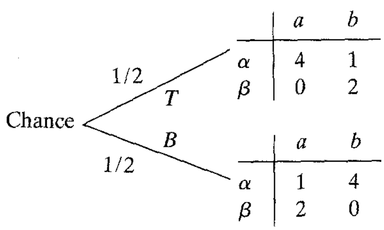
The paper is organized as follows. Section 2 introduces the idea of communication in a game tree through an experiment. It is shown that this communication is facilitated by the existence of an easily accessible general principle of behavior. The corresponding approach is formalized Section 3. Complexity of operationalizing a general principle of forward induction is related to the fact that this operationalization is generated by the common knowledge of the singularities of the game tree to be played and the communication that results. In particular, simple examples show that generic solution concepts cannot exploit these singularities. In the last section, the contrast between this approach and the more traditional Bayesian approaches to extensive games is briefly summarized.

2. COMMUNICATION THROUGH THE GAME TREE: SOME EXPERIMENTAL EVIDENCE

When subjects are instructed to play a game tree without any preplay communication it is rather exceptional that they spontaneously play a

Nash equilibrium. Either there are multiple equilibria or they are hard to compute. They may also be rather counter-intuitive such as in the repetition of the prisoner's dilemma game. The players are left with the difficult task which consists to communicate through the game tree. But such communication is so constrained that moves are hard to interpret and meaningless things happen. Yet, in some circumstances, such communication spontaneously takes place. This is the case in the following examples that may be viewed as illustrations of decentralized decision making in a team (Marshack and Radner, 1972).

Consider the simple interaction procedure associated with the following game (cf. Example 1).



Example 1.

- Player 1 knows the outcome of the chance move, either T with probability 1/2 or B with probability 1/2, then he selects a move either α or β,
- Player 2 knows player 1's move but not the outcome of the chance move, he selects a move either a or b depending on player 1's move,
- The players payoffs are identical,
- Moreover assume that once the chance move has been made the corresponding sub-game it repeated n times, yet the total payoff are only revealed at the end.

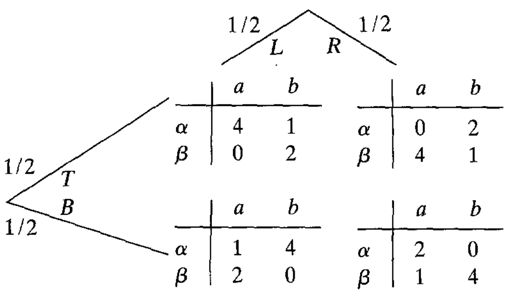
It is a simple experimental exercise to check that without preplay communication many subjects will easily find a way to communicate and to obtain the conditional Pareto outcome whatever the outcome of the chance move. Common knowledge of the rules of the games is exploited. This common knowledge enables the players to assume that no error is in principle admissible: any behavior has some cause since by convention it has been agreed upon to play as if the game was real.

Here is a sequence of the reasoning that usually happens:

- Player 1 realizes that he should signal his position but this is impossible in a one stage game, he plays α at the first stage,
- Player 2 realizes that he should be aware of player 1's signal, yet no signal would be meaningful at the first stage, he plays anything, say a,
- Observing 'αa' makes a reference, this history would be a 'good' history in state T but not in state B, accordingly player 1 second move is α if T and β if B,
- Player 2 understands the signal and adjusts accordingly.

What makes a history 'good' or 'bad' is intuitive. Formally the only thing that matters is that the first moves should be relevant but they could be relevant in different ways, that is why it is difficult to formalize. The players could have a complicated view on what is good or bad yet, as long as they have the same, it does not matter. If the game is long enough and if they have a different initial view on good or bad they will eventually have time to readjust.

Consider now Example 2.



	a	b
α	4	1
β	0	2

	a	b
α	0	2
β	4	1

	a	b
α	1	4
β	2	0

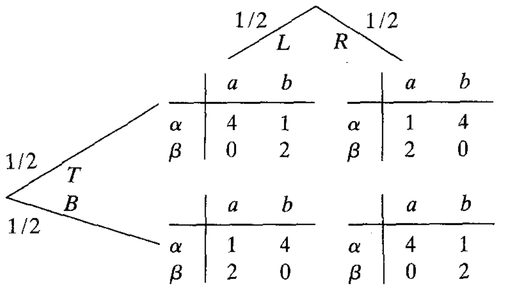
	a	b
α	2	0
β	1	4

Example 2.

- Player 1 observes T or B and player 2 observes L or R;
- Again they play in sequence the game selected by chance for n successive stages and the payoffs are only revealed at the end.

It is experimentally interesting to check that sophisticated subjects usually have no difficulty to coordinate on this game as long as they recognize that they should initiate the game by any move and then make sense of this history.

It would be false to think that tacit coordination is always feasible even for such a simple class of games. Here is a counter example (cf. Example 3).



Example 3.

In this example there seems to be no intuitive way to make sense of a history and in an experiment the subjects usually get completely discouraged after some time.

What makes coordination feasible or not? This question can be related to the problem of focal points as discussed in Shelling (1960) as well as to the problem of selection of Nash equilibria. Whereas the former approach has remained largely qualitative, the latter approach has become quite mathematically involved (Kohlberg and Mertens, 1986).

This paper adopts a somewhat median approach referred to as interactivity. It is characterized by the following four steps:

- An economic game tree is only a model not raw data, as such it can be embedded into a family of game trees which more or less characterizes the same underlying economic situation (on the contrary, a parlor game is a dead model which only survives because of its complexity);
- Prior to playing a specific game tree the players get involved into a preplay communication phase to discuss general principles that should apply to the family of game trees;
- At the beginning of the game tree to be played the players are independently told a reference plan which suggests a particular path to be played. Yet the players are not committed to this plan in the sense that if one of them unilaterally deviates and this

deviation is compatible with the general principles then the reference plan should not be implemented;
- The theoretical objective is to identify general principles, and reference plans that are self enforceable according to these principles, and this for each family of game trees.

Before illustrating this approach a comment is appropriate. The idea of plan as introduced here is different from the notion of strategy as defined by von Neumann and Morgenstern (1953). A strategy refers to an individual who seeks to decide in advance what to do under any contingency that may arise. A plan refers to a sequence of moves which lead to a play of the game tree. As such it may be compatible with many individual strategies. Moreover, a plan is a collective notion and indeed this approach to extensive games has some cooperative ingredients just as the very notion of equilibrium or focal point already has.

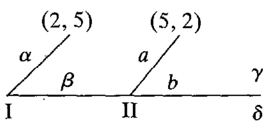
3. DESIGNING INTERACTIVE PLANS FROM GENERAL PRINCIPLES: SOME ILLUSTRATIONS

This section consist of three parts. In the first part the idea that the approach only makes sense when a particular family of game trees is defined will be discussed. Then a somewhat trivial illustration will be presented. Lastly a more interesting case related to entry situations is analyzed in detail.

3.1. A Preliminary Example

Consider the following game tree and its normal form counterpart (cf. Example 4)

$$\begin{array}{c} \\ \alpha \\ \beta\gamma \\ \beta\delta \end{array} \begin{array}{c} \begin{array}{ccc} a & bc & bd \end{array} \\ \begin{bmatrix} 2,5 & 2,5 & 2,5 \\ 5,2 & 4,3 & 0,0 \\ 5,2 & 3,0 & 1,4 \end{bmatrix} \end{array}$$



$$\begin{array}{c} c \quad\;\; d \\ \begin{bmatrix} 4,3 & 0,0 \\ 3,0 & 1,4 \end{bmatrix} \end{array}$$

Example 4.

- Player 1 selects α or β, if α is selected the game ends with payoffs (2, 5) otherwise,
- Player 2 selects a or b, if a is selected the game ends with payoffs (5, 2) otherwise,
- A last simultaneous stage is played with payoffs given by the corresponding matrix.

A first discussion of this game may be made by deriving the set of perfect Nash equilibria:

- (2, 5) with player 1 playing α and player 2 playing b and then d.
- (5, 2) with player 1 playing β and responding to b with γ with probability 4/7 and δ with probability 3/7, player 2 playing a,
- (4, 3) with player 1 playing β and responding by γ to b, player 2 playing b and then c,
- observe that the outcome (1, 4) is not a perfect Nash equilibrium since player 1's maxmin is 2, nevertheless it is a sub-game perfect equilibrium.

Then this discussion may be pursued by adopting the forward induction principle as proposed by Van Damme [1987]. This principle is stated as follows: "an equilibrium is good if no player can unambiguously signal through a deviation a better one for himself that is viable". This principle has not yet been formalized, just as Shelling's approach, it is only suggestive. Nevertheless it can be used in this example to argue that only the plans leading to (5, 2) and (4, 3) are "good" that is self enforceable with respect to an intuitive view of forward induction.

- It is immediate to see that (4, 3) is indeed good since any unilateral deviation from it at any stage is self defeating;
- Suppose (5, 2) were adopted then the only potential deviation can come from player 2 playing b instead of a. But the remaining sub-game has two sub-equilibria which generate more than 2 for player 2 namely (4, 3) and (1, 4). Thus Van Damme's principle is not satisfied;
- Suppose (2, 5) were adopted then a deviation from player 1 from α to β apparently leads to ambiguity since (5, 2) and (4, 3) are both viable in the remaining sub-game. But it can be argued that now that player 1 has signaled his intention to obtain more than 2 a 'b' move by player 2 can only result in a further coordination on

(4, 3) which is better than (5, 2) for player 2. In otherwords, given player 1's deviation, (5, 2) is no longer viable which in turn eliminates that ambiguity and invalidates the proposed plan. Player 1's signal opens the way for player 2's signal which is only feasible because of player 1's earlier deviation.

The reader will judge for himself whether or not this application of forward induction is legitimate or too much involved to remain valid. The only conclusion that will be drawn from this example is that such a way of reasoning requires so much common knowledge of the game tree and of the forward induction principle that this common knowledge has to be previously built through some form of preplay communication. But to avoid tactical considerations at this stage, the players should operate on a family of games, taking a somewhat neutral standpoint, and being only interested in designing general principles of behavior.

This seems a natural way to avoid communication of credible threats because there is mutual benefit to remain coordinated even if this may result in 'fait accompli'. If these 'fait accompli' remain consistent with the shared general principles of behavior it is only natural that their initiator should benefit from them. In a way only another 'fait accompli' can defeat a 'fait accompli'. Now prior to playing the game, it is clear that no 'fait accompli' should theoretically be feasible. This generates a never ending search for general principles that can be associated with more and more generic classes of extensive form games.

This approach is quite different from the assumption that there may be renegotiation rounds during the play (see for instance Munier and Egea, 1988, or Farrell and Maskin, 1987) since end effects prior to a renegotiation round might have unclear consequences on the analysis. It seems easier to assume that once the actual game starts only formal communication through the extensive game is meaningful.

3.2. Pure Coordination Games

This class of games refers to Shelling's focal point analysis. Consider the following game given in normal form (cf. Example 5) and suppose it is repeated n times, n being large.

$$\begin{array}{cc} & \begin{array}{cc} a & b \end{array} \\ \begin{array}{c} \alpha \\ \beta \end{array} & \begin{bmatrix} 1,1 & 0,0 \\ 0,0 & 1,1 \end{bmatrix} \end{array}$$

Example 5.

Now embed this game tree into all game trees that may be generated from any finite pure coordination game being repeated for a long period of time. A general principle of behavior to play such games might be to only be satisfied with a move if it coordinates well with the other player's choice, otherwise keep seeking. That such a simple principle may spontaneously emerge has been illustrated in Section 2. When restricted to games of complete information, such as in Example 5, this principle invalidates many plans which are consistent with traditional perfect equilibrium, and even with strategic stability, such as to always randomize with equal probabilities. Moreover it justifies a deviation from a plan even if there would be ambiguity according to a direct application of forward induction.

Consider for example a plan in which the players in sequence coordinate on (α, a) and then randomize with equal probabilities. This plan is part of a perfect equilibrium since it consists of sub-game equilibria. The only stage to make an observable deviation is when (α, a) is to occur, yet if the corresponding loss can be compensated by a viable sub-equilibrium, given the symmetry of the game, it can be compensated by more than one. If subjects were simultaneously proposed both the general principle of behavior and the plan, it seems likely that they would deviate. They would consider that there is no real ambiguity, or at least hope so, and the corresponding definition of forward induction should be amended to reflect this analysis. Can this be done independently of the context?

It is the confidence that both the game tree and the general principle of behavior are common knowledge or at least accessible, that makes a deviation from a proposed plan meaningful. In a way this is similar to Nash's original idea (Nash, 1951) except that the general principles of behavior discussed here are partly suggested by the game itself. This is clearly the case for this study of pure coordination games in the sense that the proposed principle is already stated as a coordination principle. It is suggested that this may possibly always be the case. This point of view is only accepted because of the complexity associated with any attempt to fully operationalize the forward induction principle to any kind of game tree.

3.3. *Simple Entry Games*

This complexity will be apparent through the discussion of the following simple entry games (cf. Example 6)

$$\begin{array}{cc} & \begin{array}{cc} NE & E \end{array} \\ \begin{array}{c} NE \\ E \end{array} & \begin{bmatrix} 0, 0 & 0, a \\ a, 0 & -b, -b \end{bmatrix} \end{array} \quad a > b > 0$$

Example 6.

This game is played simultaneously and repeatedly for n stages. Assume for technical simplicity that only pure strategies are feasible. Then a general principle of behavior that should guide the analysis is that "entry can only occur if it is worthwhile". When repetition is limited to two stages, this principle is equivalent to forward induction in the sense that it eliminates two successive entries by the same player. Good plans are plans in which the players alternate. Indeed:

- suppose the plan $(E, NE) \to (E, NE)$ is proposed then a deviation of player 2 results in an immediate payoff of $-b$ instead of 0, but this loss can unambiguously be compensated by the unique sub-equilibrium (NE, E) in the remaining part of the game (recall that $a > b$).

The complexity arises from the fact that forward induction does not appear to be simply generalizable to more than two stages whereas intuitively, just as for the pure coordination games, there should exist simple rules that give alternate entry without imposing this right-the-way but making it a consequence of implicit communication through the game tree.

A possible set of rules is the following. It proceeds through a formal definition of non-ambiguity.

DEFINITION OF INTERACTIVE PLANS. An interactive plan is a path in the game tree that is self enforceable given the following definition of non-ambiguity.

DEFINITION OF NON-AMBIGUITY. If the game tree has no sub-game then the plan can be embedded into a Nash equilibrium.

If the game tree has sub-games then any plan in a corresponding sub-game that can be derived from the original plan is interactive.

If after a deviation there is only one interactive plan in the corresponding sub-game that can more than compensate the deviating player then there is no ambiguity.

If there are more than one interactive plan but if there is an immediate move for the deviating player such that whatever this immediate stage conditional payoff, there remains only one interactive plan afterwards then there is no ambiguity.

If there are more than one interactive plan but if there is only one immediate move for the deviating player, that is all potentially good interactive plans start with the same move, and if any future ambiguity can be further resolved then there is no ambiguity.

PROPOSITION. *For the proposed class of simple entry games, alternate entry plans are the only interactive plans.*

Proof. Consider first a game repeated only twice, it has already been shown that repeated entry by the same player is not a good plan as a direct consequence of forward induction. Clearly alternate entry plans are self enforceable since no player can achieve a superior payoff through a unilateral deviation. What about the following two plans:

$$(NE, NE) \rightarrow (NE, E)$$

$$(E, E) \rightarrow (NE, E)$$

Observe that the definition of interactivity only implies that the last stage component of a plan be a sub-game equilibria. Thus the fact that these plans cannot be supported by any perfect equilibrium is not in itself contradictory with the definition. Yet a deviation of player 1 in any of these two plans at the initial stage is non-ambiguous. Indeed after that move there are two interactive plans consistent with the deviation, (E, NE) and (NE, E), but by playing NE player 1 obtains a conditional payoff so that his deviation is validated whatever player 2 does given that after that stage the game is non-ambiguous since it comes to an end. Technically, the definition of non-ambiguity has a minmax ingredient that is used to eliminate such plans. Consequently only alternate entry plans are interactive in a two stage game.

Consider now the game repeated 4 stages and test the plan $(E, NE) \to (E, NE) \to (E, NE) \to (NE, E)$. The total associated payoffs are $(3a, a)$. Suppose player 2 deviates and incurs an immediate loss of b, since $a > b$, the only interactive plans that can compensate this loss are those in which player 2 enters at least twice and since interactive plans alternate in the last two stages this means he has to enter now:

$$(NE, E) \to (NE, E) \to (E, NE)$$

$$(NE, E) \to (E, NE) \to (NE, E)$$

Since player 2's immediate move is unique (i.e. to play E) the argument to show non-ambiguity can proceed in spite of the multiplicity of interactive plans. At the second stage before the end of the game, player 2 can play NE which generates a zero payoff for himself whatever player 1's move, but conditional on this zero payoff, there will be a unique way for him to end the game and still compensate his loss (i.e. to play E). The whole argument satisfies the definition of non-ambiguity.

The reader will verify that this reasoning can be implemented whatever the length of the game (recall that an interactive plan can only consist of interactive plans, this backward induction property greatly simplifies the construction). ■

At this point it may seem unclear whether the proposed definition, in spite of its generality, does or does not involve too much *ad hoc* ingredients suggested by a sneaking willingness to justify alternate entry as the unique solutions. In fact the only willingness is to operationalize some general principle ("entry can only occur if it is worthwhile") along the lines of forward induction and in the context of a given class of games.

It is interesting to note that the same approach applied on a different class of entry games (involving continuous strategies interpreted as prices) generates a different result namely: the same incumbent remains the whole game yet its initial price converges to average cost when the length of the game goes to infinity (Ponssard, 1989). The corresponding definition of non-ambiguity is slightly different from this one but not contradictory, it just happens that the structure of the game requires other technicalities to be analyzed. What makes the difference in the result comes from the very different ability to communicate in the two classes of games: only two possibles moves at each stage in the present case compared with a continuous range of moves in the second case. That such a structural difference implies different results is not unreasonable.

Now the interesting feature of this approach is that the technical details used to operationalize the general principles can be kept aside until the actual game is played as long as the players agree that they should play 'interactively' and take the initial plan only as a reference. Such players never consider the others as fools or temporarily absent minded. They rationalize what happens. This makes the approach very practical to design actual planning procedures in firms (Ponssard and Tanguy, 1989).

4. BAYESIAN VERSUS INTERACTIVE DECISION MAKING

The approach developed in this paper can be best understood in contrast with Bayesian decision making (Harsanyi, 1968).

A Bayesian decision maker is never surprised. He has a complete model of the universe and uses updating of prior probabilities to describe his current state of beliefs. On the contrary, an interactive decision maker is operating under two kinds of pressure: first being surprised by a move of another player, second seeking a signaling deviation that would improve his own reference payoff. These surprises, when they occur, generate a complete reshuffling of beliefs yet without ambiguity given shared general principles. In theory, errors are excluded and this decision maker does not know what to do in face of errors.

The contrast has to be emphasized. This approach to extensive games is a theory of collective rationality, there is no point in being rational alone. In a Bayesian approach individual rationality is maintained using beliefs that may constantly be deceived, i.e. generate zero probability events that actually occur. In a way interactive plans may be seen as a tentative step to develop cooperative game theory in a dynamic framework whereas perfect equilibrium is a tentative step to extend individual decision making to game situations.

Interactive processes, in spite of their apparent sophistication, generate simple principles of behavior and plans which may be interpreted as conventions (Lewis, 1969). Conventions can be defended with limited common knowledge using bounded rationality arguments and conventions of higher order. Then it is possible to accept that players agree on conventions and on a limited model of their future interaction making the whole process worthwhile and credible. This process will be worthwhile because once agreed upon conventions greatly facilitate collective production, it will be credible because there are opportunities to change conventions that is, nobody should indefinitely feel committed to a convention. This in turn justifies the search for self enforceable plans.

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Ecole Polytechnique,
Laboratoire d'Econométrie,
1 Rue Descartes,
75005 Paris,
France