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Forward Induction and Sunk Costs Give Average Cost Pricing*

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This paper applies the idea of forward induction to a classical economic problem: the existence of an efficient form of competition in the case of increasing returns to scale. It proves that, properly formalized, this idea leads to average cost pricing as the horizon goes to infinity. *Journal of Economic Literature* Classification Numbers. © 1991 Academic Press, Inc.

1. INTRODUCTION

In the recent literature on Nash refinements, the notion of forward induction appears as an attractive but rather elusive idea (Kohlberg, 1989). As compared to solution concepts based on credible threats, such as perfect equilibria (Selten, 1975), forward induction seems closer to the original idea of self-enforceability (Nash, 1951) since the very notion of threat requires some form of commitment which here seems eliminated.

This whole issue of self-enforceability and commitment is a major one as soon as the subgames have multiple perfect equilibria. Perfection does not impose any further rationality constraint and this implicitly means that the players can jointly commit themselves to any subgame perfect equilibrium. Some of these commitments may appear irrational.

One way to deal with this issue is to introduce the idea of renegotiation proofness (Maskin and Moore, 1988): a joint commitment out of the equilibrium path should punish the deviating player without hurting the punishing one. Yet this also involves a form of joint commitment on the kind of subgame perfect equilibria that can be used as credible threats.

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Forward induction approaches this issue from a different angle. Informally stated it says that once selected, a path should be immune to any deviation that could be interpreted as a valid signal. A valid signal is such that it determines without ambiguity a more profitable path to the deviating player in the remaining part of the game, this path being itself immune to a further signal. Intuitively the idea of forward induction does not seem to require any form of commitment out of the selected path. Thus it seems closer to self-enforceability.

However, forward induction is elusive, that is, difficult to formalize. Van Damme (1989) provides a number of stimulating examples but his approach remains quite open. Still he suggests that the definition of Kohlberg and Mertens (1986) is too restrictive to capture the full potential of the original idea.

The major trend of research on forward induction concentrates on signaling games and refinements of sequential equilibria (Kreps and Wilson, 1982). It tries to remain as general as possible. There is another direction that can be pursued to formalize forward induction. It is related to the notion of focal points (Shelling, 1960) which builds on the singularities of a game. For an exploratory discussion of this idea see Ponssard (1990). This paper provides a complete illustration of the approach. It defines a class of games and the associated formalization of forward induction. The formalization relies on general rationality properties, the power of which largely depends on the class of games under consideration.

The class of games to be studied consists of finite repetition of a two-player simultaneous move game with complete information. The game models the problem of entry in a market large enough to support only one firm, given the fixed cost involved. In this framework it is proved that an appropriate formalization of forward induction leads to intense potential competition in the sense that the incumbent price converges to average cost pricing as the number of repetition increases. This result favorably compares with a perfect equilibrium approach or a renegotiation proofness approach in which any individually rational outcome could be obtained.

Section 2 discusses the formalization of forward induction that will be used throughout the paper. Section 3 introduces the economic model and gives the result and the corresponding proof. Section 4 discusses the relevance of the approach.

2. A Formalization of Forward Induction for a Class of Repeated Games

As already mentioned, the notion of forward induction is difficult to formalize. No attempt is made here to propose a formalization that could be applied to any game in extensive form.

Instead, a three-step approach is used: first define a class of games in extensive form, then define forward induction and apply it to the class of games under study, finally discuss whether or not the proposed formalization involves ad hoc ingredients which invalidate the claim that the notion of forward induction is enough to obtain the result. The third step is kept until the last section.

The class of games under study (Γ) takes the form of finite repetitions (G_n) of a constituent game (G). Moreover, the constituent game is characterized as follows.

DEFINITION 1 (The constituent game G). (i) Only two-player simultaneous move games of complete information are considered;

(ii) players are restricted to only play pure strategies.

This precisely defines the class of games Γ. For any $G_n \in \Gamma$, G_n is the n-times repetition of G. Let $p \in P$ and $q \in Q$ be the moves in G of player 1 and player 2, respectively. A path in G_n is denoted as

$$(P_n, Q_n) = ((p_n, q_n), (p_{n-1}, q_{n-1}), \ldots, (p_1, q_1)) = ((p_n, q_n), (P_{n-1}, Q_{n-1})).$$

Observe that 1 is the last period. Denote by $\pi_1(P_n, Q_n)$ and $\pi_2(P_n, Q_n)$ the players' respective payoffs in G_n associated with a path (P_n, Q_n).

The proposed solution concept associates to any game G_n a set of paths S_n.

Any path $(P_n, Q_n) = ((p_n, q_n), (P_{n-1}, Q_{n-1}))$ in S_n satisfies three conditions: perfection, backward induction, and forward induction.

Condition 1 (Perfection). (P_n, Q_n) is a perfect Nash equilibrium path of G_n (in the simple sense of subgame perfection).

Condition 2 (Backward induction). If $(P_n, Q_n) \in S_n$ then $(P_{n-1}, Q_{n-1}) \in S_{n-1}$.

The third condition relies on a form of independence with respect to irrelevant alternatives. It is the key element of the formalization and requires several definitions.

DEFINITION 2 (Restriction of G). G^* is said to be a restriction of G if P is restricted to some $P^* \subset P$ (or Q is restricted to some $Q^* \subset Q$).

DEFINITION 3 (Ambiguity of a restriction of G). Consider a restriction G^* of G. If iterated elimination of weakly dominated moves in G^* yields a unique payoff outcome independent of the order of elimination, G^* is not ambiguous. Otherwise it is ambiguous.

DEFINITION 4 (Sunk cost). Consider a path $(P_n, Q_n) = ((p_n, q_n), (P_{n-1}, Q_{n-1}))$.

An initial move $p_n^c \neq p_n$ is said to involve a sunk cost for player 1 (and similarly $q_n^c \neq q_n$ for player 2) if and only if the payoffs in G associated with (p_n, q_n) and (p_n^c, q_n) are different for player 1. This difference, which may be positive or negative, is called a sunk cost and is denoted $c_1(p_n, p_n^c)$. By definition

$$c_1(p_n, p_n^c) \neq 0.$$

DEFINITION 5 (Admissible paths in G_{n-1} after a sunk cost). Assume the deviating player is player 1. Let $\pi_1(P_n, Q_n)$ be player 1's payoff in G_n and $c_1(p_n, p_n^c)$ be the sunk cost generated by the deviation p_n^c. A path (P'_{n-1}, Q'_{n-1}) in G_{n-1} is admissible if and only if

$$c_1(p_n, p_n^c) < \pi_1(P'_{n-1}, Q'_{n-1}) - \pi_1(P_{n-1}, Q_{n-1}).$$

DEFINITION 6 (Almost equivalent paths). A subset of paths S^*_{n-1} of S_{n-1} in the game G_{n-1} are said to be almost equivalent if and only if they only differ at some stage k, that is, for all (P'_{n-1}, Q'_{n-1}) and (P''_{n-1}, Q''_{n-1}) in S^*_{n-1} and for all m, $1 \leq m \leq n - 1$ and $m \neq k$:

$$(p'_m, q'_m) = (p''_m, q''_m).$$

DEFINITION 7 (Ambiguity of an admissible set of paths). Assume the deviating player is player 1. Let S^*_{n-1} be the admissible set of paths in G_{n-1} for a given path (P_n, Q_n) and a deviation p_n^c involving a sunk cost. S^*_{n-1} is not ambiguous if and only if one of the following two statements holds:

(i) S^*_{n-1} consists of a unique path;

(ii) S^*_{n-1} consists of almost equivalent paths and if G^* is the restricted game in which player 1's moves are restricted to belong to the paths of S^*_{n-1} for the stage k at which these almost equivalent paths may differ, G^* is not ambiguous.

Otherwise S^*_{n-1} is ambiguous.

Condition 3 (Forward induction). If a path is in S_n then, whatever an initial deviation involving a sunk cost, one of the following two statements holds:

(i) there is no path in S_{n-1} that is admissible,

(ii) the subset S^*_{n-1} of all admissible paths which belong to S_{n-1} is ambiguous.

This formalization of forward induction is consistent with Van Damme's original idea based on uniqueness and viability (Van Damme, 1989). Namely, if S^*_{n-1} contains only one path it is not ambiguous; this refers to uniqueness. Then, combining Conditions 2 and 3 implies that Condition 3 has to be satisfied at all stages; this refers to viability.

The proposed formalization does not require much in terms of rationality. The stringent constraint is to admit that a deviation may commit the deviating player in the future but this form of commitment is strictly limited by the definition of ambiguity.

Here is a simple example that illustrates the definition. Let G be

$$\begin{array}{c c} & \begin{array}{ccc} \alpha & \beta & \gamma \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{pmatrix} (0,4) & (1,5) & (1,3) \\ (1,0) & (0,0) & (1,3) \\ (3,1) & (0,0) & (0,0) \end{pmatrix} \end{array}$$

Recall that the players are restricted to pure strategies.

(i) For G_1, it is clear that any Nash equilibrium meets Conditions 1, 2, and 3. Thus,

$$S_1 = \{(c, \alpha); (a, \beta); (b, \gamma)\}.$$

(ii) For G_2, observe that any combination of one stage equilibria generates a perfect equilibrium path.

Consider a path such as $((a, \beta), (a, \beta))$.

Player 1's total payoff is 2, a deviation from a to b generates a sunk cost of 1, and there is a unique subequilibrium in G_1 that makes this deviation profitable, namely (c, α). Condition 3 is not satisfied since S_1^* contains a unique element.

Consider now a path such as $((c, \alpha), (c, \alpha))$.

Player 2's total payoff is 2, and a deviation of player 2 generates a sunk cost of 1. Thus $S_1^* = \{(a, \beta); (b, \gamma)\}$. Given that there is no subsequent play, all admissible paths are almost equivalent paths.

G^* is the game

$$\begin{array}{c c} & \begin{array}{cc} \beta & \gamma \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{pmatrix} (1,5) & (1,3) \\ (0,0) & (1,3) \\ (0,0) & (0,0) \end{pmatrix} \end{array}$$

Using iterated elimination of weakly dominated moves in G^* results in the singleton (a, β) independent of the order of elimination. Accordingly G^* is not ambiguous and neither is S_1^*. Condition 3 is not satisfied.

Consider the path $((b, \gamma), (b, \gamma))$.

Observe that a deviation of player 1 from b to a does not generate a sunk cost, as such it cannot be used as an argument to invalidate such a path. However, a deviation from b to c can be used and indeed invalidates this path.

The reader will check that $((b, \gamma), (c, \alpha))$ does satisfy the three conditions of forward induction.

3. A Repeated Game of Entry

3.1. *The Game G*

A simple economic model is used which depends only on two parameters F and ω, the firm's strategic variables being the (nonnegative) prices p and q, respectively.

The parameter F refers to a fixed cost ($F > 0$) which is incurred only in case of strictly positive production (there is zero marginal cost). The cross elasticities of the demand functions depend on ω.

Define the functions d_i^d and d_i^m as

$$d_1^d(p, q) = 1 + \omega(q - p) - p; \qquad d_2^d(p, q) = 1 + \omega(p - q) - q$$

$$d_1^m(p) = (1 + 2\omega)(1 - p)/(1 + \omega); \qquad d_2^m(q) = (1 + 2\omega)(1 - q)/(1 + \omega).$$

The firms are required to meet demands at the prices they set, and these demands are given by d_1^d and d_2^d when both are nonnegative, by d_i^m when it is nonnegative and d_j^d is negative, and by 0 when d_1^d and d_2^d are both negative or when d_i^d and d_j^m are both negative. Clearly, d_i^m and d_i^d are identical when $d_j^d(p, q) = 0$. For a given price of its competitor each firm faces a continuous but kinked demand curve, the higher the value of ω the stronger the kink.

Observe that the unconstrained monopoly price is 0.5 and that the corresponding profit is bounded by 0.5.

Now the respective values of F and ω may be adjusted so that only one firm may make some positive profit at a time. More precisely, and for reasons to be detailed later on, two inequalities are assumed to hold:

$$\omega(1 + 2\omega)(2 + 3\omega)/(2 + 4\omega + \omega^2)^2 \leq F \leq (1 + 2\omega)/4(1 + \omega).$$

Note that the higher the value of ω the less stringent the inequalities. Table I gives a numerical example with $F = 0.35$ and $\omega = 10$.

This game may be interpreted as follows: each firm submits a price then the demand eventually splits or goes to only one firm, and finally any firm which is allocated a strictly positive demand must produce at a fixed cost F.

Now if one thinks of the economic situation as being repeated, it is intuitive that the level of the incumbent price imposes a constraint on the

TABLE I

PAYOFFS × 10^2 WITH $F = 0.35$ AND $\omega = 10$

p \ q	0.373	0.329	0.311	0.262	0.242	0.188
0.373	−11.6 −11.6	1.6 −28	5.9 0	1.7 0	0 0	−5.9 0
0.329	−28 1.6	−12.9 −12.9	−8 −18.8	1.7 0	0 0	−5.9 0
0.311	0 5.9	−18.8 −8	−13.6 −13.6	−2.8 −28.8	0 0	−5.9 0
0.262	0 1.7	0 1.7	−28.8 −2.8	−15.7 −15.7	−12.3 −20.4	−5.9 0
0.242	0 0	0 0	0 0	−20.4 −12.3	−16.7 −16.7	−9.6 −29.7
0.188	0 −5.9	0 −5.9	0 −5.9	0 −5.9	−29.7 −9.6	−21.2 −21.2

potential entrant. This constraint takes the form of a sunk cost computed as the fixed cost F minus the revenue. The lower the incumbent price the lower this revenue and the higher the sunk cost. If entry is profit motivated and rational this sunk cost must be compensated by future profits. These profits can only be obtained through operating in the market later.

Then, if one accepts the idea of forward induction, the questions to solve are the following ones:

—Given a price for the incumbent, what makes the potential entrant moves not ambiguous?

—Are there acceptable levels of prices for the incumbent so that entry is deterred because of ambiguity of all possible moves by the potential entrant?

—If there are, what is the impact of the length of the game?

—Can the firms share the market and achieve some Pareto optimal payoff vector or is that kind of tacit collusion in contradiction with the idea of self-enforceability associated with forward induction?

The claim of this paper is that forward induction is enough to obtain average cost pricing as the unique outcome as the horizon goes to infinity.

3.2. *The Game G_1*

The following notation is adopted. Define the functions $q^d(\cdot)$ and $q^c(\cdot)$ as

$$d_2^d(p, q^d(p)) = 0; \qquad d_1^d(p, q^c(p)) = 0.$$

Define $p^d(\cdot)$ and $p^c(\cdot)$ in a similar fashion. Let $\pi^m(\cdot)$ be the monopolist profit function

$$\pi^m(p) = \pi_1(p, q^d(p))$$
$$= (1 + 2\omega)p(1 - p)/(1 + \omega) - F \text{ (and similarly for } \pi^m(q)).$$

Average cost pricing is defined by p^l or q^l such that

$$\pi^m(p^l) = \pi^m(q^l) = 0 \qquad \text{with } p^l \text{ and } q^l \in [0, \tfrac{1}{2}].$$

This notation should be easy to keep in mind if one remembers the following. The two firms cannot both be in the market and make positive profits. Suppose that at each stage there is an incumbent and a potential entrant. Let firm 1 be the incumbent and firm 2 be the entrant:

p^a stands for a potentially "acceptable" price for firm 1;

q^d stands for a (high) price for firm 2 at which it "drops" from the market. More precisely, it is the lowest price at which it will sell zero. Thus it is obtained through $d_2^d(p^a, q^d(p^a)) = 0$;

q^c stands for a price for firm 2 at which it defects from q^d and "calls" the price p^a, in other words it sets a low price, enters given p^a, and generates a sunk cost relative to q^d; more precisely q^c is such that this sunk cost is the lowest one, and it is obtained through $d_1^d(p^a, q^c(p^a)) = 0$ (using the best response curve and standard calculus this statement is equivalent to the assumption $F \geq \omega(1 + 2\omega)(2 + 3\omega)/(2 + 4\omega + \omega^2)^2$);

p^l stands for average cost pricing; it is the limit to which p^a is claimed to converge as n goes to infinity (the existence of p^l is obtained by the assumption $F \leq (1 + 2\omega)/4(1 + \omega)$).

Going back to the numerical example, one gets: if $p^a = 0.262$ then $q^d = 0.329$ and $q^c = 0.188$. As for $p^l = q^l$, it is 0.242. Figure 1 gives a graphical representation.

Using this notation, one may express the set of pure Nash equilibria in G_1 as two symmetric line segments $\{(p^a, q^d(p^a))\}$ and $\{(p^d(q^a), q^a)\}$ such that

(i) $p^l \leq p^a \leq p_1^1$ and $q^l = q^c(p_1^1)$;
(ii) $q^l \leq q^a \leq q_1^1$ and $p^l = p^c(q_1^1)$.

(For simplicity it is assumed that p_1^1 and q_1^1 are less than 0.5; otherwise the upperbound of p^a and q^a is 0.5.)

In other words, only one firm operates and may generate some profit. The potential entrant equilibrium price is not indeterminate; it must deter further price increase on the part of the incumbent. The two symmetric line segments are depicted Fig. 2.

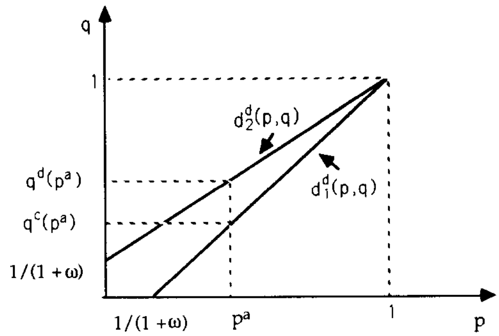

FIG. 1. A potentially acceptable price p^a and the associated dropping price p^d and calling price p^c.

Although an analysis using best response curves may be used to obtain this result, the following argument may also be satisfactory. Take a point such as (p^l, q_1^1). This is an equilibrium as long as (i) $q^c(p^l)$ involves a positive sunk cost, which is true since otherwise p^l could not possibly be average cost pricing, and (ii) p^l is a best response to q_1^1, which is true since by definition $p^l = p^c(q_1^1)$, thus p^l generates the lowest sunk cost relative to q_1^1 (in that case it does not generate any). Now, this reasoning can be extended as long as $q^c(p^a) \leq q_1^1$.

The set of equilibria S_1 of G_1 has a lot of singularities. These singularities explain the power of Conditions 1–3. Observe in particular that:

(i) S_1 can be divided into two families $\{(p^a, q^d(p^a))\}$ and $\{(p^d(q^a), q^a)\}$ and this partition is equivalent to saying which firm may eventually make some profit;

(ii) when firm 1 (and similarly firm 2) is restricted to prices in $[p, p_1^1]$ with $p^l \leq p < p_1^1$, the associated restricted game G^* is not ambiguous;

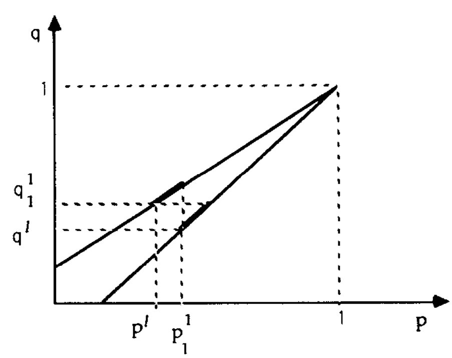

FIG. 2. The equilibria of G_1.

iterated elimination of weakly dominated moves results in the unique equilibrium outcome in which entry is most profitable for firm 1;

(iii) for any $(p^a, q^d(p^a))$ and for any $\varepsilon > 0$, there exists $p^c \neq p^a$ such that $c_1(p^a, p^c) < \varepsilon$ (and similarly for any $(p^d(q^a), q^a)$) that is, the incumbent can incur a sunk cost which may be as small as possible;

(iv) for any $(p^d(q^a), q^a)$ with $q^a \neq q_1^1$, there exists $\varepsilon > 0$ such that $\forall p^c \neq p^d(q^a)$, $c_1(p^d(q^a), p^c) > \varepsilon$ (and similarly for any $(p^a, p^d(p^a))$ with $p^a \neq p_1^1$), that is, the entrant cannot incur an arbitrarily small sunk cost.

3.3. *The Game G_2*

The game G_2 is now studied using the formalization of forward induction given in Section 2.

LEMMA 1. *In G_2 the firms cannot alternate on the market.*

Proof. Suppose it is the case that firm 1 is in the market and then firm 2. Since firm 1 is not in the market at the second stage it must be that it is in with a nonnegative profit at the first stage; otherwise the corresponding path would not be perfect. If it is in a position to make some profit it can incur a sunk cost arbitrarily small. The set of paths in G_1 that compensate this loss is a subset of $\{(p^a, q^d(p^a))\}$. In the associated restriction G^* of G, firm 1 is restricted to $p \in \,]p^l, p_1^1]$. By construction firm 2 will never make any profit; its moves are all weakly dominated by $q^d(p_1^1)$. G^* can then be reduced to a singleton and so is not ambiguous. The subset of paths to which firm 1 is restricted in G_1 is not ambiguous so that the path under consideration in G_2, in which the firms alternate, violates condition 3. ∎

LEMMA 2. *S_2 is such that either $p_1 = p_1^1$ or $q_1 = q_1^1$.*

This lemma means that the incumbent, whichever it is, has to select its most favorable subequilibrium at the last stage. The proof is omitted since it essentially amounts to repetition of the argument of Lemma 1.

To characterize S_2 it is enough to give the range of acceptable prices for the incumbent at the initial stage. Assume firm 1 is the incumbent and now define p_2^2 such that

$$\pi^m(q^c(p_2^2)) + \pi^m(q_1^1) = 0;$$

assume for the time being that $p^l \leq p_2^2 \leq p_1^1$. (For the numerical example $F = 0.35$, $\omega = 10$, since $\pi^m(q_1^1) = 0.059$ it can be computed that $q^c(p_2^2) = 0.188$ and $p_2^2 = 0.262$).

For any choice of p_2 at the initial stage which is lower than p_2^2, the minimal sunk cost that firm 2 can incur is $-\pi^m(q^c(p_2)) > -\pi^m(q^c(p_2^2))$. It cannot be compensated at the last stage since $\pi^m(q_1^1)$ represents the highest value in terms of Nash equilibrium. On the contrary, if $p_2 > p_2^2$, there is a sunk cost that can be associated with a subset of $\{(p^d(q^a), q^a)\}$ which is not ambiguous. This leads to a full characterization of S_2. Like S_1 it consists of two symmetric families of paths:

$$(P_2^a, Q_2^d) = ((p_2^a, q_2^d(p_2^a)), (p_1^1, q^d(p_1^1)))$$
$$(P_2^d, Q_2^a) = ((p_2^d(q_2^a), q_2^a), (p^d(q_1^1), q_1^1))$$

in which

$$p^l \leq p_2^a \leq p_2^2$$

with

$$\pi^m(q^c(p_2^2)) + \pi^m(q_1^1) = 0.$$

and

$$q^l \leq q_2^a \leq q_2^2$$

with

$$\pi^m(p^c(q_2^2)) + \pi^m(p_1^1) = 0.$$

The structure of S_2 is such that it can again be divided into two families. Furthermore within one family all paths are identical except at the initial stage. This singularity shows that Condition 3 of Section 2 can be very powerful in the context of these games whereas, as it stands, one would not expect much of it. Formally, the study of the game repeated three times is identical to the study of the game repeated twice. And so on.

3.4. *The Repeated Game G_n*

At this point what remains to be proved is that $p^l \leq p_2^2 \leq p_1^1$ and more generally that this remains true as the game is repeated.

This condition will ensure that it is in the short term best interest of the incumbent to deter entry. Otherwise S_n may not exist for $n \geq 2$. This condition is always satisfied as long as the two inequalities on F and ω hold.

The following simple notation is adopted (to simplify the notation further the incumbent is arbitrarily assumed to be firm 1):

$$\pi(\cdot) = \pi^m(\cdot).$$

Define p_1 by

$$p_1 = p_1^1 = p^L.$$

Recall that

$$p^L = (1 + \omega p^l)/(1 + \omega)$$

so that $p^l < p_1$. Observe also that $\pi(q^c(p_1)) = \pi(q^c(p^L)) = 0$. For $n \geq 2$, define p_n by

$$\pi(q^c(p_n)) + \sum_{j=1}^{j=n-1} \pi(p_j) = 0.$$

THEOREM 1. *The sequence (p_n) is nonincreasing and converges to p^l such that*

$$\pi(p^l) = 0.$$

LEMMA 3. *The function $p \to \pi(q^c(p)) - \pi(p)$ is nondecreasing when p belongs to $[p^l, p^L]$.*

Proof. By definition

$$\pi(q^c) = (1 + 2\omega)(1 - q^c)q^c/(1 + \omega) - F.$$

But $q^c(p) = p - (1 - p)/\omega$. Then $\pi(q^c(p)) = (1 + 2\omega)(1 - p)(1 + 1/\omega)(p - (1 - p)/\omega)/(1 + \omega) - F$ so that $\pi(q^c(p)) - \pi(p) = (1 + 2\omega)^2(1 - p)(p - (1 + \omega)/(1 + 2\omega))/\omega^2(1 + \omega)$, which is increasing for $p \leq (2 + 3\omega)/2(1 + 2\omega)$.

The interesting range for p is $[p^l, p^L]$ and so it must be shown that $p^L \leq (2 + 3\omega)/2(1 + 2\omega)$. By definition

$$p^l = p^L - (1 - p^L)/\omega$$

whereas p^l is such that

$$(1 + 2\omega)(1 - p^l)p^l/(1 + \omega) = F;$$

then p^L satisfies

$$(1 + 2\omega)(p^L(1 + \omega) - 1)(1 - p^L)/\omega^2 = F.$$

Since p^L is the smaller of the two roots of this equation and since the average of the two roots is $(2 + \omega)/2(1 + \omega) \leq (2 + 3\omega)/2(1 + 2\omega)$, this concludes the proof. ■

Proof of Theorem 1. This will be done by induction.

(i) *Step* 1: $p_2 \geq p^l$. By definition

$$\pi(q^c(p_2)) + \pi(p^L) = 0$$

so that

$$\pi(q^c(p_2)) - \pi(p^l) = \pi(q^c(p^L)) - \pi(p^L).$$

Since $\pi(q^c(p))$ is increasing in p, $p_2 \geq p^l$ is equivalent to

$$\pi(q^c(p^l)) - \pi(p^l) \leq \pi(q^c(p^L)) - \pi(p^L).$$

But $p^l < p^L$ holds so that using Lemma 3 this inequality holds.

(ii) *Step n*: if $p_{n-1} \geq p^l$ then $p_n \geq p^l$. By definition

$$\pi(q^c(p_n)) + \sum_{j=1}^{j=n-1} \pi(p_j) = 0.$$

But

$$\pi(q^c(p_{n-1})) + \sum_{j=1}^{j=n-2} \pi(p_j) = 0$$

so that

$$\pi(q^c(p_n)) - \pi(p^l) = \pi(q^c(p_{n-1})) - \pi(p_{n-1})$$

and $p_n \geq p^l$ is equivalent to

$$\pi(q^c(p^l)) - \pi(p^l) \leq \pi(q^c(p_{n-1})) - \pi(p_{n-1}),$$

which is true since $p_{n-1} \geq p^l$ is assumed to hold.

(iii) *Monotonicity of* p_n. Note that

$$\pi(q^c(p_n)) - \pi(q^c(p_{n-1})) + \pi(p_{n-1}) = 0;$$

since $p_{n-1} \geq p^l$, $\pi(p_{n-1}) \geq 0$ then $\pi(q^c(p_n)) \leq \pi((q^c(p_{n-1}))$ which gives $p_n \leq p_{n-1}$. This completes the induction argument.

(iv) (p_n) *converges to* p^l. Since (p_n) is a nonincreasing sequence bounded by p^l it converges to some limit, and so does $\pi(p_n)$ and $\pi(q^c(p_n)) = -\sum_1^{n-1} \pi(p_j)$ since $q^c(p)$ is continuous. This implies that the limit of

$\pi(p_n)$ is zero as n goes to infinity; otherwise a contradiction would arise since by construction for any n, $\pi(p_n) \geq 0$. The limit of p_n is p^l. ■

Then $\pi(q^c(p_n))$ converges to $\pi(q^c(p^l))$, the negative value of which might be interpreted as the total cumulated profit that the incumbent could obtain under this solution concept; $-\pi(q^c(p^l))$ is referred to as the monopoly rent under forward induction.

THEOREM 2. The set of pure paths S_N which satisfy the conditions of forward induction is not empty and consists of two symmetric families. The family for which firm 1 is the incumbent is

$$(P_N^a, Q_N^d) = ((p_N^a, q^d(p_N^a)), (p_{N-1}, q^d(p_{N-1})), \ldots, (p_1, q^d(p_1)))$$

in which

$$p^l \leq p_N^a \leq p_N,$$

where p_n for $n = 1$ to N is defined according to Theorem 1.

Proof. Observe that the structure of S_{n-1} consists of two families of paths which, within each family, only differ at the initial stage. Observe also that the incumbent remains the same in any path of S_{n-1}. Assume now that the firms alternate in G_n, namely that firm 1 enters first and then firm 2 remains on the market until the end (otherwise Condition 2 would be violated). Firm 1 can incur a sunk cost arbitrarily small that can only be compensated by paths in S_{n-1} in which firm 1 will be the incumbent until the end, since it has already been proved that there is no path in G_{n-1} in which the two firms would alternate. Moreover all the paths in S_{n-1}^* which compensate firm 1 are identical from stage $n-2$ until the end. Definition 4 can be used to see whether S_{n-1}^* is or is not ambiguous. At this point the proof is exactly the same as in Section 3.3. ■

For this game, this shows that in the long run forward induction and sunk costs give average cost pricing.

Recall that the set of perfect equilibria to this game as n goes to infinity is the set of all individually rational payoffs (Benoit and Krishna, 1985).

As for renegotiation proofness, a procedure such as the one described in Moreaux *et al.* (1987) can be used to obtain the same result. This procedure relies on the existence of three equilibria $(\bar{p}_1, \bar{q}_1)$, $(\bar{p}_2, \bar{q}_2)$, $(\bar{p}_3, \bar{q}_3)$ in the constituent game G such that

$$\pi_1(\bar{p}_3, \bar{q}_3) < \pi_1(\bar{p}_2, \bar{q}_2) < \pi_1(\bar{p}_1, \bar{q}_1)$$

$$\pi_2(\bar{p}_1, \bar{q}_1) < \pi_2(\bar{p}_2, \bar{q}_2) < \pi_2(\bar{p}_3, \bar{q}_3).$$

The existence of equilibria in G which satisfies these constraints is clear.

COROLLARY. The monopoly rent under forward induction goes to zero as ω goes to infinity.

Proof.

$$-\pi(q^c(p^l)) = F - (1 + 2\omega)(1 - p^l)(p^l - (1 - p^l)/\omega)/(1 + \omega).$$

But $(1 + 2\omega)(- p^l(p^l/(1 + \omega) = F$ so that $-\pi(q^c(p^l)) = (1 + 2\omega)(1 - p^l)^2/\omega(1 + \omega) \leq 2/\omega$. ■

Roughly speaking forward induction makes constestability rather robust as opposed to previous results (Farrell, 1986).

4. Interpretation of the Results

From an economic standpoint the results suggest that prior to entering a market a potential competitor goes into a calculation in which it balances the costs of entering with the potential benefits of being the new incumbent. The novelty of the calculation comes from the fact that to compute the benefits it takes into account the constraint that it will be subject to further potential competition based on the same argument, but one step further. This interpretation exhibits the circular nature of forward induction. Yet this argument is not a vicious circle when the game is assumed to be finite or discounted (for an analysis of the discounted game see Ponssard (1990).

Under this interpretation the existence of barriers to entry is not directly related to the existence of fixed costs or sunk costs in the usual sense (Baumol, 1982), but to the fact that the firms can or cannot commit themselves on the economic calculations that may or may not be carried out. If the firms can commit themselves on the kind of economic calculations that are associated with this formalization of forward induction then the existence of sunk costs is not a barrier to entry provided the time horizon is long enough. This generates a new framework for other economic applications related to limit pricing.

From a game theoretic standpoint it may be worth mentioning that this approach to forward induction assumes common knowledge of rationality at every node of the game tree (Gilboa, 1989). The two players' behaviors are intertwinned by the common knowledge assumption and this is the key element to self-enforceability.

However, there seems to be a paradox here. Why would the players commit themselves to rules that seem rational but not go further in their commitment for there mutual benefit, i.e., directly commit on a Pareto optimal outcome? A possible answer is that further commitment might inhibit the "creative destruction" of a valid signal. The players commit on rules because of their general properties in a given context and not on a particular outcome because the exact game to be played is never completely known. This interpretation confers an economic touch to forward induction which may explain its elusive character.

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