ELSEVIER

Economics Letters 50 (1996) 213–221

economics letters

Efficiency of dynamic quantity competition: A remark on Markovian equilibria

Rim Lahmandi-Ayed[a], Jean-Pierre Ponssard[a,*], David Sevy[b]

[a]*Laboratoire d'Econométrie de l'Ecole Polytechnique, 1 rue Descartes, 75005 Paris, France*
[b]*CNET/DIR/SET, 38–40 avenue du Général Leclerc, 92131 Issy les Moulineaux, France*

Received 28 June 1995; accepted 31 July 1995

Abstract

This paper extends Maskin and Tirole analysis of dynamic quantity competition in terms of Markov perfect equilibria to the case in which firms have different fixed costs. It is proved that allocative efficiency still prevails but not productive efficiency.

Keywords: Markov equilibria; Dynamic oligopoly; Efficiency

JEL classification: D4; C73

1. Introduction

This paper investigates some properties of dynamic oligopoly models as introduced by Cyert and de Groot (1970) and further developed by Maskin and Tirole (1988). These authors focus on quantity competition with large fixed costs. In such a dynamic setting, two issues deserve attention when discussing a solution concept:

(i) allocative efficiency: is the incumbent price close to its average cost?

(ii) productive efficiency: is the incumbent minimizing the industry cost function?

Maskin and Tirole (hereafter MT) analyze such dynamic oligopoly games using the notion of Markovian equilibria. Under recurrent quantity competition with large fixed costs, they show that competition between symmetric firms leads to satisfactory properties both in terms of allocative and productive efficiency whenever the commitment period is short term (i.e. the discount factor between two successive commitments is close to one). This result is contrasted with that of Eaton and Lipsey (1980) in which competition is on the timing of capital renewal. There, with short-term durable capital, productive efficiency does not hold: the optimal

* Corresponding author. Tel.: +33-4634-3241; fax: +33-4634-3428.

SSDI 0165-1765(95)00753-9

renewal policy dissipates all instantaneous rent but leads to an overaccumulation of capital. These two models can be seen as dynamic formalizations of the notion of contestability (Baumol et al., 1982). As such, they illustrate that contrasted welfare results can be obtained depending on which strategic variables are available to the firms.

Maskin and Tirole actually identify three Markovian equilibrium paths: a symmetric path and two asymmetric paths. In the asymmetric equilibrium paths the incumbent sets its monopoly quantity at each time period irrespective of how its competitor reacts; in particular, allocative efficiency is not obtained. Such equilibria are not appealing, for they do not entail the notion of short-term reaction that has been advocated by MT. In the context of symmetric firms (i.e. firms that have identical fixed costs), the symmetric equilibrium seems the most natural one to capture the dynamics of the competition.

Observe that no clear criteria, except symmetry, are introduced to select the symmetric outcome. However, this solution is not specific to a Markovian approach. It can be directly obtained through a simple rational expectation argument as in Eaton and Lipsey. More recently a similar outcome has also been obtained through a Nash refinement based on forward induction (Ponssard, 1991). In this latter case, uniqueness has been proved. On such an equilibrium path, the entry cost for the entrant is equal to the discounted rent of the incumbent. This leads to the key recursive equation of repeated games of entry with short-term commitments (Wilson, 1992) and to instantaneous rent dissipation when the discount factor goes to one (Farrell, 1986).

The objective of this paper is to detail how the symmetric Markovian equilibrium path is affected by the introduction of different fixed costs for the two competing firms. It focuses on efficiency properties when the discount factor is close to one.

It is proved that any of the two firms may remain as a permanent incumbent with a trigger strategy, i.e. committing to some optimal quantity whenever an incumbent, and entering with this same quantity if the other firm sets a quantity lower than its own trigger quantity. What was a unique equilibrium path is now subdivided into two interdependent paths. These paths are interdependent in the sense that the entry cost associated with an incumbent policy is exactly equal to the total discounted rent its competitor would obtain if it were the incumbent, and conversely. For a discount factor close to one, the optimal trigger quantities are such that the incumbent uses a policy close to its average cost pricing policy. Consequently, allocative efficiency still prevails, but productive efficiency may not since the high-cost firm might as well stay as a permanent incumbent. Furthermore, relative efficiency is not rewarded since both firms, whether high-cost or low-cost, earn exactly the same incumbency rent.

Section 2 recalls the MT framework. The results are derived in Sections 3 and 4. Connections with and differences between various approaches to dynamic oligopoly games are discussed in Section 5.

2. The model

Let us consider two firms competing in quantities. The per stage profit is defined as

$$\pi^i(q^1, q^2) = \begin{cases} (1 - q^1 - q^2)q^i - f_i\,, & \text{if } q^i > 0\,, \\ 0\,, & \text{if } q^i = 0\,. \end{cases}$$

MT introduce a constant marginal cost, which we set to zero without loss of generality. The pure monopoly revenue is $\frac{1}{4}$ and is associated with a quantity of $\frac{1}{2}$. The natural monopoly assumption amounts to the following constraints on the fixed costs: one firm is viable ($f_i < \frac{1}{4}$) but two colluding firms are not ($\frac{1}{8} < f_i$). These constraints imply that the monopoly quantity $q = \frac{1}{2}$ deters entry in the static game (it is the Stackelberg solution). In the MT model, firms are symmetric, i.e. $f_1 = f_2$.

MT consider the following infinite horizon game structure. Moves are made alternately at discrete time periods. A firm is committed to its quantity choice for two periods and can only react to its competitor's policy with a one-period lag. Firms maximize their discounted profits by a common discount factor δ. The economic analysis of dynamic competition with short-term commitments refers to the limit properties of the model when the discount factor is close to one.[1]

3. Productive efficiency of Markov perfect equilibria

The restriction to Markov equilibria consists of two conditions. For firm 1 (and similarly for firm 2),

(i) the action at time $t = 2k$ depends only on the payoff-relevant history according to the reaction function $R^1_{2k} : q^1_{2k} = R^1_{2k}(q^2_{2k-1})$, and

(ii) the stationarity of the reaction functions: R^1_{2k} is independent of the time index $2k$.

3.1. The symmetric case

Consider first the case of identical firms. The notation closely follows the MT paper,[2] except that $\bar{q}$ is used instead of q^*.

At the symmetric Markov equilibrium only one firm is in the market with a quantity $q_{\mathrm{I}} = \bar{q}_\delta$, while the entrant's optimal strategy is to stay out if $q_{\mathrm{I}} \geqslant \bar{q}_\delta$ and to enter with a quantity $q_{\mathrm{E}} = \bar{q}_\delta$ if $q_{\mathrm{I}} < \bar{q}_\delta$. The reaction function is thus:

$$R(q^j) = \begin{cases} 0\,, & \text{if } q^j \geqslant \bar{q}_\delta\,, \\ \bar{q}_\delta\,, & \text{if } q^j < \bar{q}^\delta\,. \end{cases}$$

We denote by $W^i_\delta(q^i)$ the discounted profit for firm i if it is currently committed to quantity q^i (i.e. it played q^i at the latest period prior to the current one) assuming both firms play optimally from the current period on. For $q^i = \bar{q}_\delta$, MT prove that for δ close enough to one:[2]

$$W^i_\delta(\bar{q}_\delta) = \pi^i(\bar{q}_\delta, 0)/(1-\delta) = -\pi^i(\bar{q}_\delta, \bar{q}_\delta)/\delta\,. \tag{1}$$

[1] MT also characterize the Markovian equilibria when the discount factor is not close to one. This case is not considered in this paper.

[2] Note that MT allow negative prices when the total quantity exceeds one.

This simply follows from eq. (6) in MT, which is

$$\pi(\bar{q}_\delta, \bar{q}_\delta) + \delta\pi(\bar{q}_\delta, 0)/(1-\delta) = 0 \,.$$

Eq. (1) refers to the key recursive equation of entry games with short-run commitments. It states that at the equilibrium the discounted profit of the incumbent (discounted back from one period $(\delta\pi(\bar{q}_\delta, 0)/(1-\delta))$ given the alternative move model specification) is equal to the entry cost $(-\pi(\bar{q}_\delta, \bar{q}_\delta))$. From this equation, provided that the entry cost is bounded, we readily obtain that the instantaneous rent $\pi(\bar{q}_\delta, 0)$ goes to zero as δ goes to one.

It is technically convenient to define the limiting case $\delta = 1$ as the limit of the infinite horizon discounted solution when δ converges to 1. Then, the limit $\bar{q}$ of $\bar{q}_\delta$ is the largest root of

$$(1-q)q - f = 0 \,,$$

while the limit of $W_1^i(\bar{q}_\delta)$ is equal to $-\pi(\bar{q}, \bar{q}) = -((1-2\bar{q})\bar{q} - f) = (\bar{q})^2$.

3.2. The asymmetric case

Consider now an asymmetric situation with different fixed costs, $f_1 \neq f_2$. Suppose firms 1 and 2 play according to the reaction functions R^i defined as follows:

$$R^i(q^j) = \begin{cases} 0\,, & \text{if } q^j \geqslant \bar{q}^j_\delta \,, \\ \bar{q}^i\,, & \text{if } q^j < \bar{q}^j_\delta \,, \end{cases} \tag{2}$$

with $(\bar{q}^1_\delta, \bar{q}^2_\delta)$ the largest solution of the following system of equations:

$$\begin{aligned} &\pi^1(\bar{q}^1_\delta, \bar{q}^2_\delta) + \delta\pi^1(\bar{q}^1_\delta, 0)/(1-\delta) = 0 \,, \\ &\pi^2(\bar{q}^1_\delta, \bar{q}^2_\delta) + \delta\pi^2(0, \bar{q}^2_\delta)/(1-\delta) = 0 \,. \end{aligned} \tag{3}$$

Then the following proposition and the associated corollary hold.

Proposition. In an asymmetric situation $(f_1 \neq f_2)$, *with* δ *close enough to one,* (R^1, R^2) *defined by* (2) *and* (3), *is a Markov perfect equilibrium.*

Corollary. At this Markov equilibrium, the instantaneous incumbency rent is the same whether the incumbent is the strong or the weak firm. It is equal to $(1-\delta)\bar{q}^1_\delta\bar{q}^2_\delta$.

4. Proofs of the proposition and the corollary

We provide a formal proof of the proposition for the case when δ is equal to 1. As we shall see below, simple continuity considerations ensure the result for δ close to 1.

The system (3) implies for $i = 1, 2$:

$$\bar{q}^i_\delta(1-\bar{q}^i_\delta) - f_i - (1-\delta)\bar{q}^1_\delta\bar{q}^2_\delta = 0 \,. \tag{4}$$

When δ goes to 1, the limit policies and the limit discounted payoffs can be readily characterized.

The limits $\bar{q}^i$ of the optimal trigger values $\bar{q}^i_\delta$ are the respective largest roots of

$$q^i(1-q^i)-f_i=0\,.$$

$\bar{q}^i$ is then the average cost policy for firm i.

As for $W^i_\delta(q^i)$, the discounted profit for firm i if it is committed to quantity q^i, the equivalent of (1) is written

$$W^i_\delta(\bar{q}^i_\delta)=\pi^i(\bar{q}^i_\delta,0)/(1-\delta)=-\pi^i(\bar{q}^1_\delta,\bar{q}^2_\delta)/\delta\,. \tag{5}$$

Using (4) we obtain that

$$W^i_\delta(\bar{q}^i_\delta)=\bar{q}^1_\delta\bar{q}^2_\delta\,.$$

The discounted profit is thus independent of i, because it corresponds to an instantaneous rent $(1-\delta)\bar{q}^1_\delta\bar{q}^2_\delta$.

We now prove that, if firm j plays according to R^j, then it maximizes its intertemporal profit at any time (at which it moves), assuming that henceforth both firms move according to (R^1,R^2). Without loss of generality, the incumbent is assumed to be firm 1, which can be either the weak or the strong firm.

We denote by $V^2(q^1,q^2)$ the intertemporal profit of firm 2 if it selects q^2 at the current stage, given that firm 1 is currently committed to q^1 and that henceforth both firms move according to (R^1,R^2). It will be convenient to visualize the successive moves of the two firms in a diagram.

- Suppose that firm 1 is committed to $q^1<\bar{q}^1$. We show that firm 2's best response is to enter and play $\bar{q}^2$.
  - Suppose that firm 2 plays $q^2=\bar{q}^2$. Given the reaction functions R^1 and R^2, the successive moves are depicted in Fig. 1.

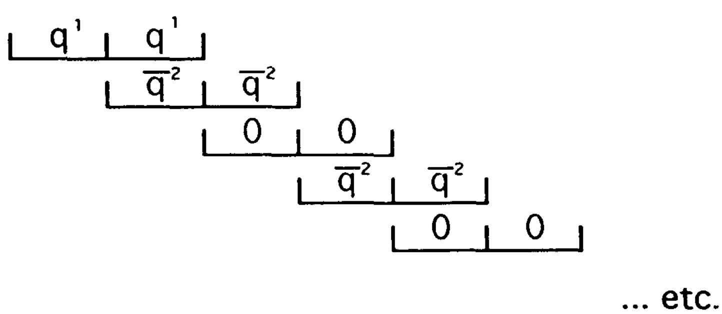

Fig. 1.

Then we have

$$V^2(q^1, \bar{q}^2) = \pi^2(q^1, \bar{q}^2) + W^2(\bar{q}^2)\,,$$

Since $W^2(\bar{q}^2) = \bar{q}^1\bar{q}^2$, $V^2(q^1, \bar{q}^2) = (1 - q^1 - \bar{q}^2)\bar{q}^2 - f_2 + \bar{q}^1\bar{q}^2$.
Since $(1 - \bar{q}^2)\bar{q}^2 - f_2 = 0$, $V^2(q^1, \bar{q}^2) = (\bar{q}^1 - q^1)\bar{q}^2 > 0$.

– Suppose, instead, that firm 2 plays $q^2 < \bar{q}^2$; the successive moves are shown in Fig. 2.

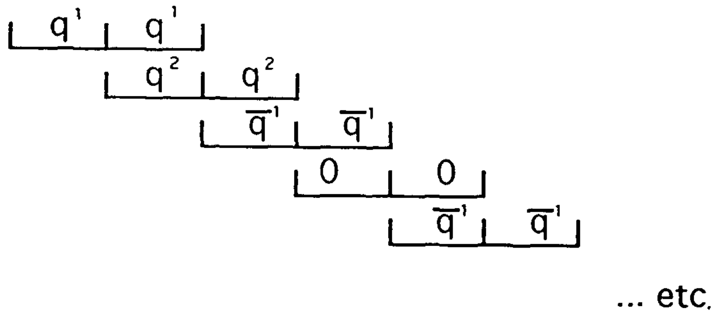

Fig. 2.

Consequently,

$$V^2(q^1, q^2) = \pi^2(q^1, q^2) + \pi^2(\bar{q}^1, q^2) = q^2(2 - q^1 - \bar{q}^1 - 2q^2) - 2f_2\,.$$

This expression reaches its maximum at the quantity

$$\hat{q}^2(q^1) = (2 - q^1 - \bar{q}^1)/4\,,$$

$$V^2(q^1, \hat{q}^2(q^1)) = (\tfrac{1}{8})(2 - q^1 - \bar{q}^1)^2 - 2f_2\,.$$

Let us define $F(q^1) = V^2(q^1, \hat{q}^2(q^1)) - V^2(q^1, \bar{q}^2)$. Then,

$$F(q^1) = (\tfrac{1}{8})(2 - q^1 - \bar{q}^1)^2 - 2f_2 - (\bar{q}^1 - q^1)\bar{q}^2\,.$$

F is a convex function, so it reaches its maximum at 0 or at $\bar{q}^1$:

$$F(0) = 2(1 - \bar{q}^1)^2 - \bar{q}^1\bar{q}^2 - 2f_2\,.$$

Since both $\bar{q}^1$ and $\bar{q}^2$ are greater than $\frac{1}{2}$, and $f_2 > \frac{1}{8}$, we obtain

$$F(0) < (2)(\tfrac{1}{4}) - (\tfrac{1}{2})(\tfrac{1}{2}) - 2(\tfrac{1}{8}) = 0$$

and

$$F(\bar{q}^1) = (\tfrac{1}{2})(1 - \bar{q}^1)^2 - 2f_2\,,$$

so that

$$F(\bar{q}^1) < (\tfrac{1}{2})(\tfrac{1}{4}) - 2(\tfrac{1}{8}) < 0\,.$$

This means that for all $q^1 < \bar{q}^1$, $V^2(q^1, \hat{q}^2(q^1)) < V^2(q^1, \bar{q}^2)$. In other words, firm 2 is strictly better off playing $\bar{q}^2$ rather than $q^2 < \bar{q}^2$.
– Suppose now that firm 2 plays $q^2 > \bar{q}^2$ (Fig. 3).

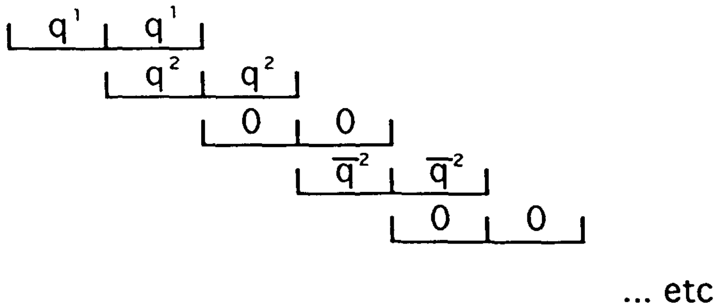

Fig. 3.

Firm 2 obtains a total profit equal to

$$V^2(q^1, q^2) = \pi^2(q^1, q^2) + \pi^2(0, q^2) + \bar{q}^1\bar{q}^2 ,$$
$$V^2(q^1, q^2) - V^2(q^1, \bar{q}^2) = [2(1 - q^2)q^2 - 2f_2] + q^1(\bar{q}^2 - q^2) .$$

Recall that $\bar{q}^2$ is the greatest root of $(1 - q)q - f_2 = 0$, which means that the first term in the preceding expression is strictly negative for $q^2 > \bar{q}^2$. The second term obviously is negative.
Consequently, $q^2 = \bar{q}^2$ is the best reaction to a quantity $q^1 < \bar{q}^1$ as long as (R^1, R^2) are anticipated in the future. Since $q^2 = \bar{q}^2$ strictly dominates the other alternatives ($q^2 < \bar{q}^2$ and $q^2 > \bar{q}^2$), the same holds true for $\bar{q}^2_\delta$ when δ is close to 1, by continuity of the different expressions.

- Suppose now that $q^1 \geq \bar{q}^1$. The diagrams depicting the successive moves are identical and so are the expressions for $V^2(q^1, q^2)$.
  – If firm 2 plays the quantity 0, then it obtains 0 as a total profit.
  – If firm 2 plays a quantity $q^2 = \bar{q}^2$, then it will obtain a profit equal to $\bar{q}^2(\bar{q}^1 - q^1)$, which is negative.
  – If firm 2 plays a quantity q^2 with $0 < q^2 < \bar{q}^2$, then it obtains at best the profit

$$V^2(q^1, \hat{q}^2(q^1)) = (\tfrac{1}{8})(2 - q^1 - \bar{q}^1)^2 - 2f_2 .$$

On $[q_1, 1]$, $V^2(\cdot, \hat{q}^2(\cdot))$ is a convex function of q^1, so it reaches its maximum at $\bar{q}^1$ or at 1:

$$V^2(\bar{q}^1, \hat{q}^2(\bar{q}^1)) = (\tfrac{1}{2})(1 - \bar{q}^1)^2 - 2f_2 < (\tfrac{1}{2})(\tfrac{1}{4}) - 2(\tfrac{1}{8}) < 0 ,$$

the last inequality being implied by $\bar{q}^1 > (\frac{1}{2})$ and $f_2 > (\frac{1}{8})$:

$$V^2(1, \hat{q}^2(1)) = (\tfrac{1}{8})(1 - \bar{q}^1)^2 - 2f_2 < (\tfrac{1}{8})(\tfrac{1}{4}) - 2(\tfrac{1}{8}) < 0$$

Consequently, the profit that firm 2 gets with a quantity $0 < q^2 < \bar{q}^2$ is strictly negative.

– If firm 2 plays a quantity $q^2 > \bar{q}^2$, then it earns

$$V^2(q^1, q^2) = 2[(1 - q^2)q^2 - f_2] + [\bar{q}^1\bar{q}^2 - q^1q^2] .$$

When $q^2 > \bar{q}^2$, the first term is negative. The second term obviously is negative since $q^1 \geqslant \bar{q}^1$ and $q^2 > \bar{q}^2$. Hence, the profit that firm 2 gets with a quantity $q^2 > \bar{q}^2$ is also strictly negative.

In summary, the quantity 0 is the best reaction to the quantity $q^1 \geqslant \bar{q}^1$. Since 0 strictly dominates the other quantities, the same result holds for δ close to 1, by continuity. □

The proof of the corollary, which states that both firms have the same incumbency rent, has already been derived as a direct consequence of system (5).

5. Discussion

It is interesting to contrast the results obtained under Markov equilibrium with the solution obtained through forward induction in finite games. Forward induction, as defined in Ponssard (1991), captures the notion of commitment by enforcing an incumbent's moves such that an entrant's entry cost could not be compensated by its future equilibrium incumbency rent. This creates a recursive construction which puts a strong competitive pressure on the incumbent policy: with symmetric firms forward induction leads to average cost pricing.

We consider now a repeated game of entry in which the two firms have different fixed costs. It can be shown that a small cost difference will generate an increasing difference in the respective incumbency rents, so that eventually a weak firm has no entry-deterring policy (Gromb et al., 1993). Then, a long finite game can be analyzed as a sequence of identical independent subgames. The number of stages in each subgame is precisely the *minimal* duration such that the weak firm cannot remain as a permanent incumbent.

When the relative cost position varies from $f_2 \ll f_1$ to $f_2 = f_1$, the duration of the subgame goes from 1 to the total length of the original game. The associated average incumbency rent for the strong firm varies from its unconstrained monopoly rent to almost zero. The average incumbency rent for the weak firm is zero at all times.

Consequently, while the Markovian solution enforces allocative efficiency but not productive efficiency under cost asymmetry, the reverse is true for the forward induction approach. In the latter case, only the most efficient firm can remain as a permanent incumbent, so that competition has a selection power. The strong incumbent enjoys some market power, which depends on its relative cost position with respect to its competitor; although distortive from an allocative standpoint, this market power rewards efficiency fairly.

Acknowledgements

The views expressed here are those of the authors and not necessarily those of France Telecom. We thank Jean Tirole for helpful comments.

References

Baumol, W.J., J.C. Panzar and R.D. Willig, 1982, Contestable markets and the theory of industry structure (Harcourt Brace Jovanovich, New York).

Cyert, R. and M. De Groot, 1970, Multi-period decision models with alternating choices as the solution of the duopoly problem, Quarterly Journal of Economics 84, 419–429.

Eaton, B. and R. Lipsey, 1980, Exit barriers are entry barriers: The durability of capital as a barrier to entry, Bell Journal of Economics 10, 721–729.

Farrell, J., 1986, How effective is potential competition? Economic Letters 20, 67–70.

Gromb, D., J-P. Ponssard and D. Sevy, 1993, Repeated games of entry with complete information and forward induction, mimeo, Laboratoire d'Econométrie de l'Ecole Polytechnique, Paris.

Maskin, E. and J. Tirole, 1988, A theory of dynamic oligopoly. Part I: Overview and quantity competition with large fixed costs, Econometrica 56, no. 3, 549–569.

Ponssard J.P., 1991, Forward induction and sunk costs give average cost pricing, Games and Economic Behaviour 3, 221–236.

Wilson, R. 1992, Strategic models of entry deterrence, in: R.J. Aumann and S. Hart, eds., Handbook of game theory, vol. 1 (Elsevier Science, Amsterdam).