ASYMMETRIES IN COST STRUCTURES AND INCENTIVES TOWARDS PRICE COMPETITION

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This paper provides some theoretical grounds to relate asymmetries in cost structures and incentives towards price competition. Typically low cost firms favor price competition whereas the reverse is true for high cost firms. Increased price competition will tend to diminish price-cost margins for all firms but the low cost firms may increase their total profits through an enlarged market share. This analysis depends on two relevant parameters: the way the overall market will react to increased price competition and interfirm cross elasticities. This is proved using comparative statics at the Nash equilibrium of an oligopolistic model.

1. Introduction

It has been generally argued that firms avoid price competition and develop non-price strategies which increase market differentiation through location, advertising, quality, etc. [see, for instance, Schmalensee (1976) for the increasing role of non-price competition]. The goal of this paper is to show that this result depends crucially on an assumption of symmetry in the cost structures. As soon as this assumption is given up different firms may have different incentives towards price versus non-price competition.

In fact the ambivalent nature of differentiation became a subject of interest in the empirical literature in the late 70's [Porter (1979), Hall (1980), Kiechel (1981)]. This opened the way to various generic strategies such as low cost combined with standardization versus differentiation oriented towards niches. The theoretical results of the model studied here can contribute to qualify these judgements. Indeed it will be shown that if non-price competition is always beneficial to high cost firms, a low cost firm faces a dilemma. This follows from the fact that more differentiation increases monopolistic prices for everybody whereas more homogeneity increases mobility in market shares and favors a low cost firm which can charge a low price. If the elasticity of the market and the advantage in cost are high enough, the
profits generated by a higher market share offset the loss incurred by a price war. These results rely on the study of comparative statics with respect to differentiation at the Nash equilibrium of an oligopoly.

In the literature differentiation has been modelled in several ways. In Lancaster's or Hotelling's approach, differentiation is seen as a product of long term strategic choices of the firms: when choosing the quality of the products, the locations of their plants, differentiation may result as an equilibrium for the firms since they so escape price wars (each firm capturing different consumers). Such models rapidly become mathematically untractable and have to multiply numerous specific assumptions; the problem is to derive demand functions and short run price equilibria for all possible long term choices of the firms [see Gabszewicz-Thisse (1979)].

Here, we have chosen another route: demand laws are specified and parameters of the model are directly interpreted in terms of elasticities and differentiation. In particular the parameter of differentiation is seen as a parameter of 'viscosity': the more the market is homogeneous, the more the demand curve for a firm is elastic, the limit case being the competition 'à la Bertrand'. The value of this parameter is of course thought to depend on consumer's preferences, on advertising policies, etc., but this dependence is not explicit in our model. Still the model focuses on two ingredients which are essential to our problems: the cross elasticities of volume to price between competitive firms and the overall reactions of the market to increased price competition. In practice, such numerical information may sometimes be obtained at the firm level. Indeed our approach has been suggested by an empirical study of the automobile industry [Ouanes (1981)] and as such may be of some use in other sectors as well.

Section 2 presents the model. In section 3, we derive interesting general properties such as the existence and uniqueness of the equilibrium, as well as the particularly well behaved limits of the equilibrium prices when homogeneity increases. The uniqueness of equilibrium allows to make comparative statics with respect to homogeneity in section 4, paying special attention to the duopoly case.

2. The model and some general properties

2.1. Hypotheses and notation

Let \( n > 1 \) be the number of firms in the industry; firms are alike except for their production costs. The number of firms is kept fixed throughout the paper.

Assumption 1. The marginal cost of each firm is constant.

Firms are indexed by increasing costs, i.e., if \( c_i \) denotes \( i \)'s constant marginal cost, \( c_1 \leq c_2 \ldots \leq c_n \).
The market being differentiated, each firm may charge a different price; the total demand depends on these prices through a general index.

**Assumption 2.** If firm $i$ charges price $p_i$, the total demand $V$ is equal to

$$V_0 \left( \sum p_j^{-s} \right)^{1/s},$$

where $V_0$, $s$, $e$ are positive parameters.

This means that the elasticity of demand with respect to the price index,

$$p_s = \left( \sum_{j} p_j^{-s} / n \right)^{-1/s},$$

is constant equal to $-e$.

The market shares depend only on the prices and the elasticity of substitution of $i$'s market share with respect to its price $p_i$ is supposed to be constant equal to $-s$. This gives:

**Assumption 3.** Given the prices $p_j$, $j=1,...,n$, the market share $\mu_i$ of firm $i$ is equal to

$$\mu_i = p_i^{-s} / \sum_j p_j^{-s}.$$

For $e=0$, this model coincides with Case's generic model of price competition in partially differentiated markets [Case (1979)]. To introduce a market elasticity two ways are open: define Case's model over $m$ firms and study competition among a subset of firms taking the prices of the other firms as fixed (but assuming a constant cross elasticity $s$ among firms would make the partition rather dubious), or postulate an elasticity with respect to a global price index. The second route is chosen here. From Assumptions 2 and 3 it follows directly that

$$\frac{dp_s}{p_s} = \sum_j \mu_j \frac{dp_j}{p_j}.$$

Thus the change in the general price index reflects the change in prices weighted by the respective market shares. This seems particularly appropriate.

\[ \text{Since } n \text{ is fixed we do not need to specify the dependance of } V_0 \text{ on } n, \text{ it would of course be important if entry were considered.} \]
when customers are sensitive to an average price of some goods (such as cars, soft drinks, refrigerators, home computers, etc.) and then make their choice among close substitutes. Recently, Spence (1976) and Dixit and Stiglitz (1977) obtained firm specific demand functions very close to those defined above by deriving them from a representative consumer model: the differentiation came from the desirability of variety of the consumer. Our demand functions do not derive from such utility functions. They should rather be seen as an integration of a local approach assuming that global market elasticity and cross elasticity are constant. Our use of comparative statics is consistent with this local approach: firms do not optimize on differentiation but only derive incentives towards more or less price competition depending on the viscosity of the short term price equilibrium.

The demand of firm $i$ at prices $(p_j)$ is

$$V_i - V_0 \left( \frac{p_i^{-s}}{\left( \sum_j p_j^{-s} \right)^{1-e/s}} \right)$$

and its profit $\pi_i = (p_i - c_i)V_i$.

By differentiating $V_i$ with respect to $p_j$ when $j \neq i$ one gets

$$\partial V_i/\partial p_j = V_i(s-e)\left( p_j^{-s-1} \left/ \sum_j p_j^{-s} \right. \right)$$

or

$$\partial \log V_i/\partial \log p_j = (s-e)\mu_j.$$  

Since we want to study a market where products are imperfect substitutes one assumes:

**Assumption 4.** $s \leq e$.

The formula above shows that $(s-e)$ may be taken as an index of differentiation; when $s = e$ the market is completely differentiated; when $s$ (and $s-e$) becomes higher and higher demands’ behavior follows approximately Bertrand’s assumption the firm with smaller prices gets all the demand.

When the price of firm $i$ is infinitely high, its profit is equivalent to

$$V_0 \left( p_i^{-s+1} \left/ \left( \sum_{i \neq i} p_j^{-s} \right)^{1-1/s} \right. \right).$$

Thus if $s \leq 1$ profit of each firm rises infinitely with its price (other prices

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2It is interesting to note that Selten’s approach to price competition with ‘demand inertia’ is similar in spirit to ours except that his model is linear whereas ours is exponential [Selten (1965)].
being fixed) and there would surely not exist Nash equilibria so our last assumption is:

**Assumption 5.** $s > 1$.

### 3. Existence and uniqueness of the equilibrium

We assume that firms fix their prices so as to maximize their profits, treating the prices of other firms as fixed. Thus an equilibrium is a set of prices $p_i^*, i = 1, \ldots, n$, where $p_i$ maximizes $\pi_i(p_i, p_{-i})$ on $R^+$.

We prove here that an equilibrium exists if and only if

\[(C.1) \quad n > (s-e)/(s-1)\]

and that when it exists it is unique.

These results are remarkable for their simplicity and the uniqueness of the equilibrium allows us to make some comparative statics. The condition (C.1) is effective only when $e$ is smaller than 1: it gives a lower bound on the number of active firms for an equilibrium to exist when the demand is inelastic. As it is clear from the proof when (C.1) is false an equilibrium fails to exist because firms have too much incentive to charge high prices. Such a situation is highly unstable because entry will surely occur. This argument is actually still true when $n = 1$: condition (C.1) gives then $e > 1$ which is the very condition for the existence of a finite monopoly price. (Remark that since fixed costs are not considered we have no upper bound for $n$.)

#### 3.1. Characterization of a Nash equilibrium

We say that $p_i^*$ is a best response to the prices $(p_j)_{j \neq i}$ if $p_i^*$ maximizes on $R^+$ the profit function $(p_i - c_i) V_i(p_i, p_{-i})$; $p_i^*, i = 1, \ldots, n$, is an equilibrium if $p_i^*$ is a best response to $p_{-i}^*$ for each firm $i$. We prove in the appendix the following property:

**Proposition 1.** Firm $i$ has a unique best response to $(p_j)_{j \neq i}$ characterized by

\[
(p_i > c_i, \quad (BR_i) \quad p_i/(p_i - c_i) + (s - e) \left( p_i^{-s} \left( p_i^{-s} + \sum_{j \neq i} p_j^{-s} \right) \right) - s = 0, \]

or shortly

\[
p_i/(p_i - c_i) + (s - e) \mu_i - s = 0.
\]
This best response function depends of \((p_j)_{j \neq i}\) only through the aggregate \(\sum_{j \neq i} p_j^{-2}\) and is a continuous decreasing function of it. When \(e > 1\) it takes its values between \(c_i(s/(s-1))\) and \(c_i(e/(e-1))\); when \(e \leq 1\) it is always above \(c_i(s/(s-1))\) but not bounded.

Since the monopoly price of \(i\) is \(c_i(e/(e-1))\) when \(e > 1\) and \(+\infty\) when \(e \leq 1\) the best response approaches it by below when \(\sum_{j \neq i} p_j^{-2}\to 0\), i.e., when \(p_j\to\infty\) for each \(j \neq i\). Moreover, when the market is totally differentiated \((s = e)\) the best response is constant equal to the monopoly price. From Proposition 1 one gets:

**Lemma 1.** Condition (C.1) is necessary to the existence of an equilibrium.

Let \((p_i)_i\) be an equilibrium; from Proposition 1 \(p_i/(p_i - c_i) = s - (s - e)\mu_i\) for each \(i\); adding up these equations one gets

\[
\sum_i p_i/(p_i - c_i) = (n-1)s + e; \quad p_i > c_i \quad \text{implies} \quad \sum_i p_i/(p_i - c_i) > n,
\]

and this gives condition (C.1)

### 3.2. Existence of an equilibrium

A fixed point of the map \(\phi\) from the product space \(X_i [c_i + \infty]\) in itself where \(\phi_i(p)\) is the best response to \(p_i \) is an equilibrium. From Proposition 1, \(\phi\) is continuous so the existence of an equilibrium is ensured (Brower’s theorem) if one may find some numbers \(a_i, a_i > c_i\) such that \(\phi(X_i [c_i, a_i]) \subset X_i [c_i, a_i]\). When \(e > 1\) one knows that \(\phi_i(p)\) is bounded above by \(c_i(e/(e-1))\) so it suffices to take \(a_i = c_i(e/e - 1)\).

We exhibit now some numbers \(a_i\) when the condition (C.1) is true even if \(e \leq 1\). Indeed, suppose \(n < (n-1)s + e\), there is an \(n\)-tuple \((\eta_i)_i\), \(\eta_i > 0\), \(\sum \eta_i = 1\) such that \(1 < s - (s - e)\eta_i\) for each \(i\). For any positive number \(k\), the market shares corresponding to the prices \((k\eta_i^{-1/s})_i\) are exactly \(\eta_i\). Since

\[
\lim_{k \to \infty} k\eta_i^{-1/s}/(k\eta_i^{-1/s} - c_i) = 1,
\]

one may choose \(k\) sufficiently high such that

\[
a_i/(a_i - c_i) < s - (s - e)\eta_i \quad \text{where} \quad a_i = k\eta_i^{-1/s}.
\]

This implies (see Proof of Proposition 1) that \(a_i\) is higher than the best response to \((a_j)_{j \neq i}\).
Take now \( p \) in \( X_i [c_i, a_i] \). Then \( \sum_{j \neq i} P_j^{-s} \geq \sum_{j \neq i} a_j^{-s} \) and since \( \phi_i \) is decreasing in \( \sum_{j \neq i} P_j^{-s} \) one has \( \phi_i(p) \leq \phi_i(a_i) \leq a_i \) and finally \( \phi(X_i [c_i, a_i]) \subseteq X_i [c_i, a_i] \). So we have proved:

**Proposition 2.** An equilibrium exists if and only if \((C.1)\) holds.

### 3.3. Uniqueness of an equilibrium

We prove now that the equilibrium is necessarily unique. Let \((p_i)\) and \((p'_i)\) be two distinct equilibria. Since

\[
\sum_i p_i/(p_i - c_i) = \sum_i p'_i/(p'_i - c_i),
\]

\( p'_i \leq p_i \) for each \( i \) is impossible, thus \( \alpha = \max_i p'_i/p_i > 1 \). Let \( j \) be such that \( p'_j/p_j = \alpha \) and denote by \( \mu_i \) and \( \mu'_i \) the market shares corresponding to \( p \) and \( p' \),

\[
\mu'_j = p'_j^{-s} \left/ \sum_i p'_i^{-s} \right. \geq \alpha^{-s} p_j^{-s} \left/ \sum_i p_i^{-s} \right. = \mu_j.
\]

But we know that

\[
p_j/(p_j - c_j) = s - (s - \varepsilon) \mu_j,
\]

and

\[
p'_j/(p'_j - c_j) = s - (s - \varepsilon) \mu'_j.
\]

So

\[
p_j/(p_j - c_j) - p'_j/(p'_j - c_j) = (s - \varepsilon) (\mu'_j - \mu_j),
\]

but the term on the right of the inequality is positive \( (\mu'_j \geq \mu_j) \) and the term on the left is strictly negative since \( p_j < p'_j \) which gives the desired contradiction.

### 3.4. Comparing prices, market shares and profits at equilibrium

We study here how costs affect prices, market shares and profit rates at equilibrium [the parameters are fixed and satisfy \((C.1)\)]. Recall that firms are ordered by increasing costs.

**Proposition 3.** At equilibrium, prices are increasing in costs, market shares, profit rates and profits decreasing in costs. Formally if \((p_i), (\mu_i), (\tau_i), (\pi_i)\) are the corresponding values one has: \( p_1 \leq \cdots \leq p_n, \mu_1 \geq \cdots \geq \mu_n, \tau_1 \geq \cdots \geq \tau_n \) and \( \pi_1 \geq \cdots \geq \pi_n \).
This result is highly desirable and its failure would have shed some doubt about the relevance of the model.

Let \( i,j \) be two firms with \( i < j \) (\( c_i \leq c_j \)). \( BR_i \) and \( BR_j \) are satisfied so that

\[
p_i/(p_i - c_i) - p_j/(p_j - c_j) = (s-e)(\mu_j - \mu_i).
\]

(1)

Suppose \( p_j < p_i \) then \( \mu_j > \mu_i \); but \( c_i \leq c_j \) and \( p_j < p_i \) simultaneously imply \( p_i/(p_i - c_i) < p_j/(p_j - c_j) \) and with (1) one gets \( \mu_j \leq \mu_i \), a contradiction, so \( p_i \leq p_j \) and \( \mu_i \geq \mu_j \). The profit rate \( \tau_i \) of \( i \) is equal to

\[
(p_i - c_i)V_i/p_iV_i = (p_i - c_i)/p_i.
\]

If \( i < j \) we just proved \( \mu_j \leq \mu_i \) and from (1) this implies \( 1/\tau_i \leq 1/\tau_j \) or \( \tau_i \geq \tau_j \). As for the profits, remark that

\[
\pi_i/\pi_j = (p_i - c_i)V_i/(p_j - c_j)V_j = (\tau_i/\tau_j)(p_i/p_j)^{-s+1},
\]

so if \( i < j \) \( \tau_i \geq \tau_j \) and \( p_i \leq p_j \) implies \( \pi_i \geq \pi_j \) (by Assumption 5 \( s > 1 \)).

4. Comparative statistics with respect to homogeneity

We are now interested in varying the parameter of homogeneity \( s \) and interpreting the respective impact on profits as long term incentives to change the game. The impact on prices is analyzed as well. In spite of the simplicity of the model, only partial results could be derived.

To evaluate the role of asymmetry in cost structures, the symmetric case will be studied first. In this case profits and prices are decreasing functions of \( s \).

Then the duopoly case is discussed. We prove that the price and profit rate of firm 2 (with higher cost) is always decreasing in \( s \). However, we find a U-shape function for the price and profit of firm 1. This is complemented by a numerical example in which the profit exhibits the same behavior.

In case of an arbitrary number of firms we could only succeed in the study of the limiting values of profits and prices when the market becomes more and more differentiated (\( s \to e \)) or more and more homogeneous (\( s \to +\infty \)).

4.1. The symmetric case

When all the marginal costs are equal, the equilibrium is easy to compute: by the uniqueness property of equilibrium all the prices must be equal and thus the market shares equal to \( 1/n \). So if \( p(s) \) is the common equilibrium price one gets from Proposition 1:
\[
\frac{p(s)}{p(s) - c} = s - \frac{s - e}{n} \quad \text{or} \quad p(s) = c \left( 1 + \frac{1}{s - 1 - \frac{s - e}{n}} \right)
\]

[(C.1) is assumed, i.e., the term \( s - 1 - (s - e)/n \) is positive]. Thus \( p(s) \) is a decreasing function of \( s \) as well as the profit \( \pi(s) = V_0n^{-e}(p(s) - c)(p(s))^{-e} \). Indeed \( \pi'(s) = V_0n^{-e}p(s)[1 - e((p(s) - c)/p(s))] \); the term in brackets being positive \( \pi'(s) \) is negative so the firms have no incentives towards increased price competition.

4.2. The duopoly case

We study here the case \( n=2 \). Only prices and profit rates could be treated analytically.

In Appendix B, we prove the two following propositions:

**Proposition 4.** The price \( p_2(s) \) and the profit rate \( \tau_2(s) \) are decreasing in \( s \) for the high cost firm.

**Proposition 5.** The price \( p_1(s) \) and the profit rate \( \tau_1(s) \) are first decreasing and then increasing in \( s \).

We could not show that firm 1's profit is a U-shape function but the example we have treated numerically suggests that it is the case (see table 1 for a numerical example).

4.3. Limiting values of the model in the general case

One easily proves the results presented in table 2. When the market is

<table>
<thead>
<tr>
<th>( s )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>15</th>
<th>20</th>
<th>30</th>
<th>( \infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_1 )</td>
<td>10</td>
<td>8.956</td>
<td>8.378</td>
<td>8.031</td>
<td>7.810</td>
<td>7.664</td>
<td>7.363</td>
<td>7.371</td>
<td>7.437</td>
<td>8.1</td>
</tr>
<tr>
<td>( \mu_1 )</td>
<td>0.719</td>
<td>0.736</td>
<td>0.760</td>
<td>0.783</td>
<td>0.805</td>
<td>0.824</td>
<td>0.914</td>
<td>0.938</td>
<td>0.962</td>
<td>1</td>
</tr>
<tr>
<td>( \mu_2 )</td>
<td>0.281</td>
<td>0.264</td>
<td>0.240</td>
<td>0.216</td>
<td>0.195</td>
<td>0.176</td>
<td>0.086</td>
<td>0.062</td>
<td>0.037</td>
<td>0</td>
</tr>
<tr>
<td>( v )</td>
<td>0.0140</td>
<td>0.0153</td>
<td>0.163</td>
<td>0.0171</td>
<td>0.176</td>
<td>0.0180</td>
<td>0.0187</td>
<td>0.0185</td>
<td>0.0181</td>
<td>0.0150</td>
</tr>
<tr>
<td>( \pi_1 )</td>
<td>0.050</td>
<td>0.0445</td>
<td>0.0420</td>
<td>0.0406</td>
<td>0.0399</td>
<td>0.0395</td>
<td>0.0403</td>
<td>0.0412</td>
<td>0.0425</td>
<td>0.0468</td>
</tr>
<tr>
<td>( \tau_1 )</td>
<td>0.50</td>
<td>0.44</td>
<td>0.40</td>
<td>0.38</td>
<td>0.36</td>
<td>0.35</td>
<td>0.32</td>
<td>0.32</td>
<td>0.33</td>
<td>0.27</td>
</tr>
<tr>
<td>( \pi_2 )</td>
<td>0.0312</td>
<td>0.0186</td>
<td>0.0125</td>
<td>0.0088</td>
<td>0.0065</td>
<td>0.0049</td>
<td>0.0010</td>
<td>0.0005</td>
<td>0.0002</td>
<td>0</td>
</tr>
<tr>
<td>( \tau_2 )</td>
<td>0.50</td>
<td>0.37</td>
<td>0.28</td>
<td>0.23</td>
<td>0.19</td>
<td>0.16</td>
<td>0.07</td>
<td>0.05</td>
<td>0.03</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1

A numerical example for the duopoly case with \( c_1 = 5, c_2 = 8, e = 2 \).
Table 2

Limiting values of the model as function of differentiation.

<table>
<thead>
<tr>
<th>Case</th>
<th>( s )</th>
<th>( p_1 )</th>
<th>( p_{n+1} )</th>
<th>( \mu_1 )</th>
<th>( \mu_i )</th>
<th>( \nu )</th>
<th>( \pi_1 )</th>
<th>( \tau_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( s = e ) (if ( e &gt; 1 ))</td>
<td>( \frac{e}{e-1} c_1 )</td>
<td>( \frac{e}{e-1} c_i )</td>
<td>( \frac{c_1^{-e}}{\sum_j c_j^{-e}} )</td>
<td>( \frac{e}{(e-1)^{-e}} \left( \sum c_i^{-e} \right) )</td>
<td>( (e/e_1/(e-1))^{-e+1}/e )</td>
<td>( \frac{1}{e-1} )</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>( s \to \infty )</td>
<td>( \frac{e}{e-1} c_1 &lt; c_2 )</td>
<td>( \frac{e}{e-1} c_1 )</td>
<td>( c_i )</td>
<td>1</td>
<td>0</td>
<td>( \left( \frac{e}{(e-1)} c_1 \right)^{-e} )</td>
<td>( (e/e_1/(e-1))^{-e+1}/e )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>and ( e &gt; 1 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( 1 - \mu_1 \sim \left( \frac{e}{e-1} c_2 \right)^{e} )</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>( s \to \infty )</td>
<td>( \frac{e}{e-1} c_1 &gt; c_2 )</td>
<td>( c_2 )</td>
<td>( c_i )</td>
<td>1</td>
<td>0</td>
<td>( c_2^{-e} )</td>
<td>( (c_2-c_1)c_2^{-e} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>and ( e &gt; 1 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( s(1-\mu_1) \sim \frac{c_2}{c_2-c_1} - e )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>or ( e \leq 1 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
sufficiently homogeneous (i.e., $s$ high enough) the condition (C.1) is fulfilled but if one wants to consider equilibria in a market more and more differentiated one has to assume $e > 1$ since (C.1): $(n - 1)s + e > n$, is true when $s \to e$ only if $e > 1$ (moreover, by Assumption 3 $s$ is always higher than 1).

Observe that when $s \to \infty$ two cases should be considered depending on the respective position of firm 1’s monopoly price $((e/(e - 1))c_1)$ and firm 2’s marginal cost $c_2)$. Thus the limit price equilibrium corresponding to a homogeneous market deters entry and is equivalent to Bertrand’s approach.

4.4. Comments

These results justify the comments made in the introduction. As soon as the symmetry assumption is dropped, firms may have different incentives to move the game towards price versus non-price competition through various long term differentiation strategies.

The antagonistic incentives may be exemplified considering two extreme situations: $s$ close $e$ and $s$ much larger than $e$. These two situations may, respectively, be interpreted as fragmented industries (small cross elasticity) and commodity good industries (high cross elasticity). In the first one all firms share the incentive to move towards more differentiation, each one exploiting its own niche. In the second one the low cost firm has a definite incentive to an increase in homogeneity contrary to the other firms’ traditional incentive towards more differentiation. The leading firm will sufficiently increase its market share to compensate its loss on marginal revenue. This points out the crucial role of cross elasticities and relative cost position in assessing a long term strategy. As such it provides some theoretical ground to the current emphasis on generic strategies, that is, different firms pursuing different strategies in the same industry [for a discussion of the recent shift in strategic planning, see Kiechel (1981)].

Our model should be seen as one of the simplest ones in which one can discuss the role of differentiation, incorporating the minimal number of ingredients to make this discussion worthwhile: different marginal costs, global market elasticity and interfirrm elasticity. Though they are derived from a very specific and simple model, our results should be locally robust.

Still this approach is limited in many ways and one may wish to pursue the discussion either in more general models, explicitly including the available long term strategies, or on a case by case basis, explicitly including the other significant features of a specific industry.

Appendix A

Proof of Proposition 1.

A best response of $i$ to $p_{-i} = (p_j)_{j \neq i}$ maximizes $\pi_i = (p_i - c_i)V_i$. Since $V_i$ is strictly positive for every positive price a best response is surely in $]c_i, + \infty[$.
and is a zero if \( \frac{\partial \pi_i}{\partial p_i} \)

\[
\frac{\partial \pi_i}{\partial p_i} = \left( V_i/p_i \right) \left[ p_i + (p_i - c_i) (-s + (s - e) \mu_i) \right].
\]

On \( \partial \pi_i + \infty \left[ \frac{\partial \pi_i}{\partial p_i} \right] \) is of the same sign as the function

\[
\phi_i(p_i, q_{-i}) = \frac{p_i}{(p_i - c_i)} + \frac{(s - e)}{(p_i^{-s}/(p_i^{-s} + q_{-i}))} - s,
\]

where \( q_{-i} = \sum_{j \neq i} p_j^{-s} \).

Since \((s - e)\) is positive \( \phi_i \) is decreasing in \( p_i \); moreover,

\[
\lim_{p_i \to c_i} \phi_i(p_i, q_{-i}) = +\infty, \quad \lim_{p_i \to +\infty} \phi_i(p_i, q_{-i}) = 1 - s < 0.
\]

Thus \( \phi_i \) has a unique zero \( p_i^* \) on \( \partial c_i + \infty \) and \( p_i^* \) is the unique best response to \((p_{-i})\), it is determined implicitly by the equation \( \phi_i(p_i^*, q_{-i}) = 0 \) \((q_{-i} \) is the function \( \sum_{j \neq i} p_j^{-s} \) of \( p_{-i} \) but we omit the argument \( p_{-i} \) to simplify). We denote it \( p_i^*(q_{-i}) \).

The function \( \phi_i(p_i, q_{-i}) \) is continuously differentiable on \( \partial c_i + \infty \left[ q_{-i} \right] \), \( + \infty \), is decreasing in \( p_i \) and \( q_{-i} \); therefore, by the implicit function theorem, \( p_i^*(q_{-i}) \) is a continuous decreasing function of \( q_{-i} \) and its limits are

\[
\lim_{q_{-i} \to 0} p_i^*(q_{-i}) = \frac{c_i e}{e - 1} \text{ if } e > 1 \text{ or } +\infty \text{ if } e \leq 1,
\]

and

\[
\lim_{q_{-i} \to +\infty} p_i^*(q_{-i}) = \frac{c_i s}{s - 1}.
\]

Q.E.D.

Appendix B

When \( n = 2 \) the condition (C.1) is \( s + e > 2 \), we already know that \( p_i(s) \) is a smooth function of \( s \) for \( s > 2 - e \); its derivative is obtained by differentiating with respect to \( s \) the implicit equations \( (BR_i) \); this yields

\[
-\frac{c_i}{(p_i - c_i)^2} \frac{dp_i}{ds} = 1 - \mu_i(s) - (s - e) \frac{d\mu_i}{ds}, \quad i = 1, 2,
\]

where \( d\mu_i/ds \) is the derivative of \( s \to p_i(s)^{-s}/(p_i(s)^{-s} + p_2(s)^{-s}) \).

One finds

\[
\frac{d\mu_i}{ds} = -\frac{d\mu_2}{ds} - \mu_1 \mu_2 \left[ \log \frac{p_2}{p_1} + s \left( \frac{1}{p_2} \frac{dp_2}{ds} - \frac{1}{p_1} \frac{dp_1}{ds} \right) \right].
\]
Replacing these expressions in (B.1) one finds a differential system satisfied by the functions \((p_i)\). This system is not solvable and it is of no use writing it. We may derive our first result in a simple way: from (B.1) we have that

\[
\frac{dp_i}{ds} \geq 0 \iff 1 - \mu_1(s) - (s-e) \frac{d\tilde{\mu}_i}{ds} \leq 0.
\]  

(B.3)

This allows us to prove Proposition 4.

**Proof.** \(dp_1/ds\) and \(dp_2/ds\) are never simultaneously positive since by (B.3) one would have

\[
\left(1 - \mu_1(s) - (s-e) \frac{d\tilde{\mu}_1}{ds}\right) + \left(1 - \mu_2(s) - (s-e) \frac{d\tilde{\mu}_2}{ds}\right) = 1 \leq 0
\]

Suppose now \(dp_2/ds \geq 0\), then \(dp_1/ds \leq 0\) and from (B.3) one gets

\[
1 - \mu_2(s) \leq (s-e) \frac{d\tilde{\mu}_2}{ds} \leq 0.
\]

But from (B.2) if \(dp_1/ds \leq 0\), and \(dp_2/ds \geq 0\), one gets \(d\tilde{\mu}_2/ds < 0\) since one always has \(p_2(s) > p_1(s)\) (Proposition 3); this gives the contradiction and \(dp_2/ds < 0\). Since \(\tau_2 = (p_2 - c_2)/p_2\) varies as \(p_2\) this gives the result.

The sign of \(dp_1/ds\) is more complicated to determine. First, by (B.3) when \(s=e\) one gets \(dp_1/ds < 0\) and this is of no surprise since \(p_1\) tends to the monopoly price when \(s\) tends to \(e\). To go further we compute a differential equation satisfied by \(\mu_1\); from (2) one gets

\[
\frac{d\tilde{\mu}_1}{ds} - \mu_1 \mu_2 \left( \frac{1}{s} \log \frac{\mu_1}{\mu_2} + s \frac{d}{ds} \left( \log \frac{p_2}{p_1} \right) \right).
\]

Another way to write \((BR_i)\) is

\[
p_i = c_i \left(1 - \mu_1(s) - (s-e) \frac{d\tilde{\mu}_i}{ds}\right),
\]

and one gets

\[
\frac{dp_1}{p_1 ds} = \left(1 - \mu_1 - (s-e) \frac{d\tilde{\mu}_1}{ds}\right) \left[ \frac{1}{s-(s-e)\mu_1} - \frac{1}{s-(s-e)\mu_1 - 1} \right],
\]

and a similar expression for \(dp_2/dp_1\). Using the fact that \(d\tilde{\mu}_2/ds = -d\tilde{\mu}_1/ds\) and setting \(\delta_i = s-(s-e)\mu_i\) one finally obtains
\[
\frac{d\mu_1}{ds}\left[\left(1+s(s-e)\mu_1\mu_2\left(\frac{1}{\delta_1-1} - \frac{1}{\delta_1} + \frac{1}{\delta_2-1} - \frac{1}{\delta_2}\right)\right]\right]
\]

\[
= \mu_1\mu_2\left[\frac{1}{s}\log \frac{\mu_1}{\mu_2} + \cdots \right]
\]

\[
\cdots + s(1-\mu_1)\left(\frac{1}{\delta_1-1} - \frac{1}{\delta_1}\right) - s(1-\mu_2)\left(\frac{1}{\delta_2-1} - \frac{1}{\delta_2}\right)\].
\] (B.4)

Remark that \(\delta_i\) is higher than 1 since \(p_i/(p_i-c_i) = \delta_i\), so the term multiplying \(\delta\mu_1/\delta s\) is positive.

This equation is independent of \(c_1\) and \(c_2\), so this means that the trajectory of \(\mu_1\) depends on \(c_1\) and \(c_2\) only through the 'initial' conditions, i.e., when \(s = e\mu_1(e) = c_1^{-e}/(c_1^{-e} + c_2^{-e})\).

We want now to show that \(p_1\) is a U-shape function, i.e., is first decreasing and then increasing; the fact that \(p_1\) is first decreasing has been proved just above. We claim now that \(dp_1/\delta s\) has one unique zero and \(\lim_{s \to \infty} dp_1/\delta s \geq 0\) and this will prove the U shape.

First \(dp_1/\delta s\) has at most one zero; indeed by (B.3)

\[
\frac{dp_1}{ds} \geq 0 \Leftrightarrow 1 - \mu_1(s) - (s-e)\frac{d\mu_1}{ds} \leq 0,
\] (B.5)

which is equivalent by (B.4) and the remark below (3.4) to

\[
1 + s(s-e)\mu_1\mu_2\left(\frac{1}{\delta_1-1} - \frac{1}{\delta_1} + \frac{1}{\delta_2-1} - \frac{1}{\delta_2}\right) \leq (s-e)\mu_1
\]

\[
x\left[\frac{1}{s}\log \frac{\mu_1}{\mu_2} + s(1-\mu_1)\left(\frac{1}{\delta_1-1} - \frac{1}{\delta_1}\right) - s(1-\mu_2)\left(\frac{1}{\delta_2-1} - \frac{1}{\delta_2}\right)\right],
\]

that is,

\[
1 \leq \frac{(s-e)}{s} \mu_1\left[\log \frac{\mu_1}{\mu_2} - s^2\left(\frac{1}{\delta_2-1} - \frac{1}{\delta_2}\right)\right].
\] (B.6)

Denote by \(g(s)\) the function in the right side of (B.6). Let \(s_0\) be a zero of \(dp_1/\delta s\); then \(g(s_0) = 1\); suppose that \(g\) is increasing near \(s_0\) then \(1 \leq g(s)\) [respectively \(1 \geq g(s)\)] is true for \(s\) near \(s_0\) when \(s \geq s_0\) (respectively \(s \leq s_0\)) thus [by the equivalence between (B.5) and (B.6)] \(p_1\) is decreasing before \(s_0\) and increasing after \(s_0\).

We prove now that for every zero of \(dp_1/\delta s\) it is the case that \(g\) is locally increasing; this implies the existence of at most one zero.
So let \( s_0 \) be a zero of \( dp_1/ds \); by (B.5) \((s-e)d\bar{\mu}_1/ds = 1 - \mu_1(s)\) so \( \mu_1 \) is increasing. This implies that

\[
 s \rightarrow s - \frac{e}{s} \mu_1 \log \frac{\mu_1}{\mu_2}
\]

is increasing [we use here the fact that \( \log(\mu_1/\mu_2)(s_0) > 0 \)].

The function

\[
 s \rightarrow s^2 \left( \frac{1}{\delta_2 - 1} - \frac{1}{\delta_2} \right) = -1 \left( \frac{\delta_2}{s} - 1 \right) \left( \frac{\delta_2}{s} \right)
\]

is also increasing near \( s_0 \). Indeed the derivative of

\[
 \frac{\delta_2}{s} = 1 - \left( 1 - \frac{e}{s} \right) \mu_2 \quad \text{is equal to} \quad - \frac{e}{s^2} \mu_2 - \left( 1 - \frac{e}{s} \right) d\bar{\mu}_2/ds,
\]

and so this is equal to

\[
 - \frac{e}{s^2} \mu_2 + \frac{s_0 - e d\bar{\mu}_1}{s_0} \frac{d\bar{\mu}_2}{ds}(s_0) = \frac{\mu_2(s_0)}{s_0} \left( - \frac{e}{s_0} + 1 \right),
\]

which is positive. Thus \( s \rightarrow \delta_2/s \) is increasing near \( s_0 \) as well as \( s \rightarrow (\delta_2/s - 1/s)(\delta_2/s) \) (we use the fact that \( \delta_2 \geq 1 \)) and finally \( g(s) \) is increasing near \( s_0 \).

For the existence of such a zero we use (B.6): one knows that

\[
 \lim_{s \to \infty} \mu_2(s) = 0 \quad \text{thus} \quad \lim_{s \to \infty} \left( \frac{s-e}{s} \right) \mu_1 = 1, \quad \lim_{s \to \infty} \log \frac{\mu_1}{\mu_2} = +\infty
\]

and

\[
 \lim s^2 \left( \frac{1}{\delta_2 - 1} - \frac{1}{\delta_2} \right) = \lim \left( \frac{-1}{\left[ \left( 1 - \frac{1}{s} \right) - \left( 1 - \frac{e}{s} \right) \mu_2 \right] \left( 1 - \left( 1 - \frac{e}{s} \right) \mu_2 \right)} \right) = -1,
\]

so finally (B.6) is surely satisfied when \( s \) increases since the term on the right side tends to \( +\infty \) when \( s \to \infty \).

Summarizing these results:

**Proposition 5.** \( p_1(s), \tau_1(s) \) are first decreasing and then increasing. Q.E.D.

This is in agreement with the intuition that the firm with lowest cost has not necessary interest in a more homogeneous market but that a position in
"the middle" is surely not in its interest: if \( s^* \) gives the minimum of \( p_1 \), its market share increases \[ (d\mu_1/ds)(s^* > 0) \] with \( s \) as well as its price.

References


