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THE STRATEGIC ROLE OF INFORMATION ON THE DEMAND FUNCTION IN AN OLIGOPOLISTIC MARKET*

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This paper investigates the incentives for cooperation in market surveys among competitive firms. The analysis relies on a game theoretic model. The main conclusion is that the value of information in a competitive market exhibits a sharp decrease as the number of firms that share the information increases. Thus, the advantage obtained through sharing the cost of a market survey may be upset by the loss due to the spreading of information among the competitors. The consumer's point of view is also studied showing that market surveys are advantageous in terms of consumer surplus, a usual indicator of the consumer's satisfaction. (MARKETING; GAMES/GROUP DECISIONS; INFORMATION SYSTEMS)

1. Introduction

In recent years, there has been a growing interest in studying explicitly the role of information in economic situations. For example, Hirschleifer [6] and Marshall [10] investigated the social and private value of information in general equilibrium models in which there are uncertainties on the initial endowments. There has also been interest in some specific models including production (Arrow [1], Green [4]). In many such investigations, the strategic analysis of the role of information remains incomplete, in particular, it may not be specified whether uninformed agents are aware of the information acquisition obtained by others and may adapt to the new situation, or, the possible spreading of information (which may involuntarily take place through the informed agents' activity) may be ignored thus over-evaluating the value of private information. In a strategic analysis of information performed in a game theoretic framework, such factors may appear essential and could give rise to many counter-intuitive results such as bluff, signalling, negative value of information, controlled transmission of information and so on (Aumann and Maschler [3], Ponssard [12], Levine and Ponssard [8]). The corresponding game models belong to the class of games with incomplete information introduced by Harsanyi [5].

The subject of this paper is to study the strategic role of information in a partial equilibrium economic situation using as a model a game with incomplete information. Though the emphasis was not on the role of information, this class of games has already been used in the economic literature (d'Aspremont and Gerard-Varet [2], Wilson [13]). This paper may be seen as an attempt to use such models for bringing out specific insights about the role of information, e.g., market research.

The situation to be studied concerns an oligopolistic market for an homogenous product, the demand for which is uncertain. The questions of interest concern the incentives for firms to acquire private information. This will be shown to strongly depend on the number of firms who do so. The consumer's point of view will also be taken into account so that one may discuss the social value of information. This situation is modeled as a simple oligopolistic market game with linear demand and cost functions. The advantage of this model is that it is mathematically tractable (and it may not be an overstatement to say that games with incomplete information are complex due to the fact that a strategy is not a number but a function of the state of information). The drawback is of course that the results are constrained by the narrow assumptions.

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2. The Model and Its Solution

2.1. *Assumptions*

Consider the following simple oligopolistic market model for an homogenous product:

The *demand function* is linear, if p denotes the price, q the quantity, D and a two parameters, we have:

$$p = D - aq. \tag{1}$$

The parameter D *is a random variable* and $F(\cdot)$ is its distribution function. This may be interpreted as if a random perturbation with mean zero were added to a known demand function. Uncertainties of other types such as multiplicative shall not be considered here. The function $F(\cdot)$ is assumed to be known to the firms. The questions of interest are about the expected values of perfect information on the random parameter D. (Throughout this note, taking an expected value will be represented by the letter E, and a variance by Var so that ED stands for mean of D and Var D for the variance of D.)

There are N *competing firms*, $i = 1, \ldots, N$. The quantity produced by firm i, denoted x_i, is taken as its decision variable. The production functions, though different, will all be assumed to be linear, c_i denoting firm i's marginal cost.

The firms are maximizing their expected profit, no risk aversion will be taken into account. The resulting game is a *non cooperative game* with simultaneous moves. The solution concept to be used is the *Nash equilibrium* (Nash [11]), also called the Cournot solution in this context.

2.2. *Solutions*

2.2.1. *Solution of the no information game.* The calculation to derive the equilibrium point in this case is quite standard. Letting $\Pi_i(x_i)$ denote firm i's profit, we have:

$$\Pi_i(x_i) = (p - c_i)x_i. \tag{2}$$

Substituting for p its value according to (1), and letting $q = \sum_{j=1}^N x_j$, it follows that

$$\Pi_i(x_i) = \left(D - c_i - a\sum_{j=1}^{N} x_j\right)x_i. \tag{3}$$

Taking expected values,

$$E\Pi_i(x_i) = \left(ED - c_i - a\sum_{j=1}^{N} x_j\right)x_i. \tag{4}$$

An equilibrium point consists of a vector $(x_i^*)_{i=1,\ldots,N}$ such that the expected profit of firm i is maximal at x_i^* whenever the other firms produce x_j^*, $j \neq i$. Assume that x_i^* is not a corner solution,[1] it may be obtained by writing that the derivatives of the expected profits are zero. Then, the $(x_i^*)_{i=1,\ldots,N}$ satisfy the following set of linear equations:

$$a\sum_{j=1}^{N} x_j + ax_i = ED - c_i, \qquad i = 1, \ldots, N. \tag{5}$$

This set of equations is easily solved by adding up all equations to obtain $a\sum_{j=1}^N x_j$ and then substituting to find each value of x_i.

Then, there exists a unique equilibrium point in pure strategies (which dominates any randomized equilibrium since the payoff functions are convex). It is easily

[1] This will be so long as the solution $(x_i)_{i=1,\ldots,N}$ of (5) is nonnegative. The corresponding assumptions on the parameters' values will be assumed to hold.

calculated,

$$x_i^* = \left(\left(ED + \sum_{j=1}^{N} c_j\right) \Big/ (N+1) - c_i\right) \Big/ a. \tag{6}$$

The resulting price and its expected value are respectively

$$p^* = D - \left(NED - \sum_{j=1}^{N} c_j\right) \Big/ (N+1) \tag{7}$$

and

$$Ep^* = \left(ED + \sum_{j=1}^{N} c_j\right) \Big/ (N+1). \tag{8}$$

Finally, one obtains the expected profit of each firm at the equilibrium:

$$E\Pi_i^* = \left(\left(ED + \sum_{j=1}^{N} c_j\right) \Big/ (N+1) - c_i\right)^2 \Big/ a. \tag{9}$$

2.2.2. *The I_1 informed game and the firms' optimal profits.* In this section, it is assumed that firms $1, 2, \ldots, k$ $(k \leqslant N)$ acquire perfect information on the value taken by the random parameter D whereas firms $k+1, \ldots, N$ remain uncertain about d. This fact is known to all firms. How does this situation compare with the case in which no firm is informed?

This new situation may be modelled as a game in extensive form which starts by a chance move selecting the true value d of D according to the probability distribution $F(\cdot)$. Then this value d is exclusively revealed to informed firms whereas for the others it remains uncertain. All firms know the probability distribution according to which the true value d of D is selected. Such models are called games with incomplete information (Harsanyi [5]).

Let I_1 denote the subset of informed firms whereas I_2 denotes the subset of uninformed ones. For simplicity, we shall refer to this game as the I_1 informed game. An equilibrium point of the I_1 informed game is a vector of functions $(\bar{x}_1(\cdot), \ldots, \bar{x}_k(\cdot), \bar{x}_{k+1}, \ldots, \bar{x}_N)$, in which the informed firms select the quantity to be produced as a function of the true value d. The uninformed firms select only a point; note that for them the decisions taken by the informed firms will appear as random variables since they do not know d.

One may obtain the equilibrium point in this game, writing that informed firms maximize their profit conditional on d, whereas uninformed firms maximize expected profit (Harsanyi [5]).[2]

$$i \in I_1, \quad \Pi_i(x_i) = \left(d - c_i - a \sum_{j \in I_1} x_j(d) - a \sum_{j \in I_2} x_j\right) x_i(d), \tag{10}$$

$$i \in I_2, \quad E\Pi_i(x_i) = \left(ED - c_i - a \sum_{j \in I_1} Ex_j - a \sum_{j \in I_2} x_j\right) x_i. \tag{11}$$

[2] It should be noted at this point that the present Nash equilibrium, which is to be found by solving a set of simultaneous equations, cannot be interpreted as the stable point of usual tatônnement processes. For example, in a duopoly model a usual tatônnement process considers that each firm reacts myopically to the quantity produced by its competitor by maximizing its own profit given this quantity. Under some hypotheses, this process converges to the Nash equilibrium.

In the present case, the observation of the quantity produced by informed firms would clearly reveal some information about the true value of D to the uninformed firms and this transmission of information should be taken into account. If the situation to be modelled contains a true dynamic feature, it should be explicitly introduced into the model otherwise the role of information cannot be studied in a coherent game theoretic sense.

This note should be kept in mind for any interpretations of the results of this paper in a dynamic context.

Then, we may proceed as in §2.2.1, writing that the derivatives are zero (again assuming no corner solutions). The set of equations is as follows:

For all possible d and all $i \in I_1$,

$$a \sum_{j \in I_1} x_j(d) + a \sum_{j \in I_2} x_j + a x_i(d) = d - c_i, \tag{12}$$

for all $i \in I_1$.

$$a \sum_{j \in I_1} Ex_j + a \sum_{j \in I_2} x_j + a x_i = ED - c_i, \tag{13}$$

for all $i \in I_2$.

We are now in a position to state the following results:

THEOREM 1.[3] *There exists a unique equilibrium point in the I_1 informed game, $(\bar{x}_1(\cdot), \ldots, \bar{x}_k(\cdot), \bar{x}_{k+1}, \ldots, \bar{x}_N)$. It satisfies:*

$$i \in I_1, \qquad E\bar{x}_i = x_i^*, \tag{14}$$

$$i \in I_2, \qquad \bar{x}_i = x_i^*, \tag{15}$$

in which $(x_i^*)_{i=1,\ldots,N}$ is the solution of the no information game (cf. 2.2.1). Moreover, for each realized value d of the random parameter D, $[\bar{x}_i(d)]_{i \in I_1}$ is the Nash equilibrium of the oligopoly game with complete information, the demand function of which is given by

$$p = d - a\left[q + \sum_{i \in I_2} x_i^*\right].$$

PROOF. The set of equations (12) + (13) may be solved along the following lines. One may take an expected value over all possible d for each $i \in I_1$ to obtain:

$$a \sum_{j \in I_1} Ex_j + a \sum_{j \in I_2} x_j + aEx_i = ED - c_i. \tag{12 bis}$$

Comparing (12 bis) + (13) and the set of equations (5) it follows that, if a solution exists, (14) and (15) hold. Now, the values of $\bar{x}_i(d)$ for each d may be obtained by solving the corresponding (12) in which the x_i, $i = k + 1, \ldots, N$, are replaced by their values, x_i^*. This procedure clearly defines a unique potential solution which is found to satisfy all equations.

THEOREM 2. *Comparing with the case of no information, the expected profits at the equilibrium point of the I_1 informed game are increased by* $\operatorname{Var} D/a(k+1)^2$ *for informed firms and unmodified for uninformed firms.*

PROOF. This result will be proved as follows: first it will be shown to hold if $k = N$, i.e. all firms are informed; second, this will be extended for all values of k $(1 \leqslant k \leqslant N)$.

[3] The assumption of no corner solution is crucial for this theorem and if d is allowed to vary widely, it may easily be violated. It may also be interesting to note that the mathematical structure allowing this very simple result, which is the key to evaluating the expected values of information, is as follows: take an N person noncooperative game and let u_i be the utility function of player i, if u_i has the form

$$u_i(x_1, \ldots, x_N) = f(x_1, \ldots, x_N) + Dx_i$$

in which f is quadratic and D a random parameter, then Theorem 1 holds.

Finally, this theorem makes immaterial whether the acquisition of information is made privately or secretly (Levine and Ponssard [8]).

Let $k = N$, then the expected value of perfect information on the random parameter D is the difference between the expected profits knowing d, $E\Pi_i^*(D \mid D = d)$, and the expected profits not knowing d, $E\Pi_i^*(D)$. Using result (9), and the fact that if Y is a random variable and α and β two real numbers,

$$E[\alpha Y + \beta]^2 - [E[\alpha Y + \beta]]^2 = \alpha^2 \mathrm{Var}\ Y,$$

we have:

$$E\Pi_i^*(D \mid D = d) - E\Pi_i^*(D)$$

$$= E\left(\left(D + \sum_{j=1}^{N} c_j\right)\Big/(N+1) - c_i\right)^2\Big/a - \left(\left(ED + \sum_{j=1}^{N} c_j\right)\Big/(N+1) - c_i\right)^2\Big/a$$

$$= \mathrm{Var}\ D/a(N+1)^2. \tag{16}$$

For $1 \leqslant k < N$, using Theorem 1, one sees that the expected quantity on the market remains the same so that the expected price will also be the same. Since uninformed firms produce the same quantity in the two games (no information and I_1 informed respectively) their expected profits are unmodified.[4] To evaluate the change of profits for informed firms, one may simply consider that in the corresponding games, uninformed firms are passive players with their production levels fixed at $\bar{x}_i = x_i^*$. Then the comparison is exactly the one which resulted in (16) except that N is changed in k and that, in the demand function D is replaced by $D - aq_2$ where q_2 is the aggregate constant production of I_2 firms. This latter modification has no effect on the variance operation performed to obtain (16). This completes the proof.

2.2.3. *The consumer surplus in the I_1 informed game.* In partial equilibrium models such as the one considered in this paper, consumers are passive players with no strategic choice and no explicit utility functions. Nevertheless, it may be of interest to have an idea of how they may be affected by the acquisition of information by the firms. A possible approach to acquiring such an idea is to attribute to them an aggregate utility index such as the consumer surplus and see how it varies between the two games.

THEOREM 3. *Compared to the no information game, the expected consumer surplus in the I_1 informed game is increased by*

$$k^2 \mathrm{Var}\ D/2a(k+1)^2. \tag{17}$$

PROOF. It may be useful to follow the reasoning using Figures 1, 2 and 3. Figure 1 gives the graphical definition of the consumer surplus, it is the area of the shaded triangle. Note that it may be calculated as $(d - p^*)q^*/2$, and since $p^* = d - aq^*$, it is simply $aq^{*2}/2$. In the no information game, the aggregate production is independent of the true value d; then, though the price and the demand function are modified depending on d, the consumer surplus remains exactly the same as if D were replaced by its expected value (Figure 2).

In the I_1 informed game, the aggregate production depends on the true value d through the production levels of the informed firms. Recall equation (12),

$$a \sum_{j \in I_1} x_j(d) + a \sum_{j \in I_2} x_j + a x_i(d) = d - c_i, \tag{12}$$

[4] It will be shown later that the variance of their profits decreases. This gives an indication that if risk aversion were introduced, they would enjoy a higher utility level by having the other firms being informed.

and denote by $\bar{q}(d)$ and q^* the optimal aggregate production levels in the I_1 informed game and in the no information game respectively. Using (12) one obtains (adding (12) over the subset I_1)

$$\bar{q}(d) - q^* = kd/a(k+1). \tag{18}$$

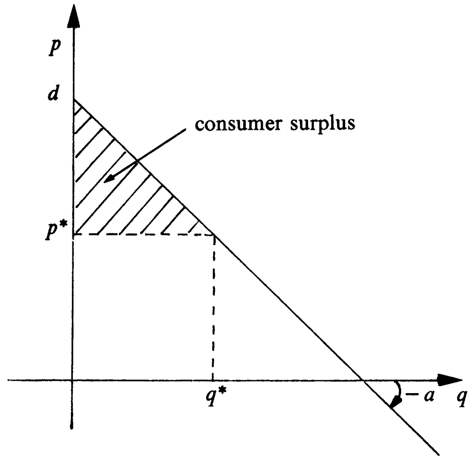

FIGURE 1. Graphical Definition of the Consumer Surplus.

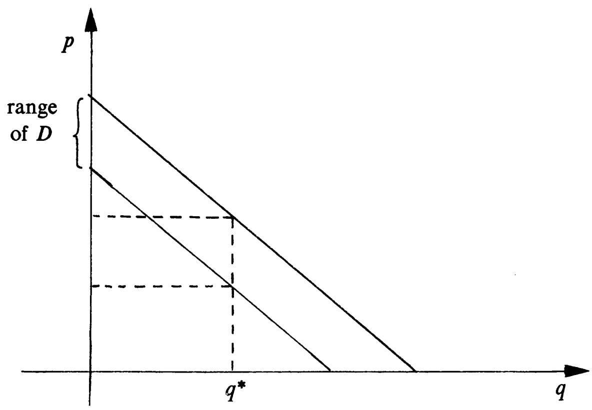

FIGURE 2. The Consumer Surplus in the No Information Game.

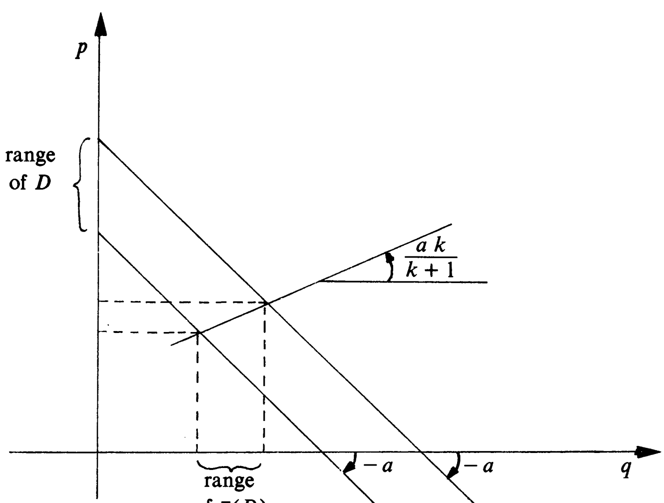

FIGURE 3. The Consumer Surplus in the I_1 Informed Game.

[Note that the variance of the equilibrium price decreases as k increases (see Figure 3); it follows that the variance of expected profits of uninformed firms decreases as well.] Since the consumer surplus is $a\bar{q}(d)^2/2$ as a function of the true value d, its expected value is $E[a\bar{q}(D)^2/2]$. Using (18) concludes the proof.

3. Discussion of the Results

The results obtained in the context of the simple model detailed in §2 allow us to initiate a discussion of the strategic role of information on the demand function. First, the firms' point of view will be discussed then, we shall turn to the consumer's point of view.

Theorem 2 indicates that there is a positive incentive for each firm to acquire private information but that this incentive strongly depends on the number of firms that will do so. Namely, this incentive decreases as $1/(k+1)^2$ where k is the number of informed firms. To develop an interpretation of this result it is interesting to make the following analogy:

—take a no information game with $N = 1$, it is a monopoly and its expected optimal profit is $(ED - c)^2/4a$; now let newcomers arrive in the market (for simplicity assume all marginal costs are equal). Then, if there are N firms altogether, each firm's optimal profit now is $(ED - c)^2/a(N+1)^2$; note that profits decrease as $1/(N+1)^2$;

—consider now the I_1 informed game, that is the oligopolistic market in which k firms know exactly the demand function; it may be seen as a two level game: at the first level, all firms compete in a market in which the demand function is the average demand function; at the second level, the informed firms, and only they, compete on the variance of the demand function; then, if there is only one informed firm it obtains a monopoly rent on the variance, that is var $D/4a$, if there are k informed firms, each obtain an oligopolistic rent of var $D/a(k+1)^2$.

In other words, there is a complete similarity between entry of new firms in a market with a known demand function and the acquisition of information by firms in a market with uncertain demand function. This gives an idea of the competitive advantage associated with the precise knowledge of the demand function. It is clear that uninformed firms have an incentive to acquire information whereas informed firms have an incentive not to share the information. In the information market, a barrier to entry is the cost of acquiring information, which may typically be a fixed cost. Then, one may expect that if this fixed cost is not too high (relative to the competitive advantage associated with information), all firms could acquire information (they may even make a syndicate to share the cost of experimentation and this collusion, provided it stops there, would be socially valuable). However, if the fixed cost is high, only a limited number of firms would acquire information until its cost equals its marginal competitive advantage for the newcomer. In this type of game, there is an advantage in moving first and the situation recalls a battle of sexes game (Luce and Raiffa [9]). An interesting question would then be to explore a dynamic market model and see if the informed firms can keep their competitive advantage over time or if there is an inevitable diffusion of information through their past moves so that their competitive advantage would decrease as time goes on (for such a dynamic analysis in a zero sum context, see Ponssard [12]).

Turning now to the consumer's point of view, Theorem 3 indicates that the consumer surplus increases when the firms are acquiring information. Moreover, it increases as the number of informed firms increases. Given the interpretation in terms of entry in the information market, this result is not surprising. It is to the consumer's interest that the firms are informed and that their number be as high as possible. But, this does not necessarily mean that if the consumer is given a strategic variable such as the possibility of revealing or not revealing the demand function, he may use a sincere strategy that always reveals his true preferences. This may be an interesting question for further study.

Taking a global point of view, one may conclude by saying that the social value of information is positive since it is positive for every economic agent. However, the preceding discussion gives certainly a more precise idea of this statement showing the partially antagonistic point of views among firms for the information market as well as between firms and consumer for the number of informed firms.[5]

[5] Detailed comments by E. Kohlberg on an earlier version of this paper were very helpful.

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